

I B. Tech I Semester Supplementary Examinations, August/Sep-2022

MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours

Max. Marks: 70

Answer any five Questions one Question from Each Unit

All Questions Carry Equal Marks

UNIT-I

1. a) Discuss the convergence of $\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{7.9} + \dots (x > 0)$. (7M)

b) Verify Lagrange's mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in $[0,4]$ and find the value of the constant. (7M)

Or

2. a) Examine the convergence of $\frac{3}{5} - \frac{5}{7} + \frac{7}{10} - \frac{9}{13} + \dots$ (7M)

b) Find Taylor's series expansion of the $f(x, y) = \cos x$ about $x = \frac{\pi}{3}$ and hence find the approximate value of $\cos 35^\circ$. (7M)

UNIT-II

3. a) Solve $(1-x^2)\frac{dy}{dx} + xy = y^3 \sin^{-1} x$. (7M)

b) A bacterial culture, increases from 100 to 400grams in 10 hours. How much would be present after 3 hours? (7M)

Or

4. a) Find the orthogonal trajectories of the following family of curves: $r^n = a^n \sin n\theta$. (7M)

b) Solve $2xydy - (x^2 + y^2 + 1)dx = 0$. (7M)

UNIT-III

5. a) Solve $(D^2 + 1)y = \sin x \sin 2x + e^x x^2$. (7M)

b) Solve $(x+3)^2 \frac{d^2 y}{dx^2} + (x+3) \frac{dy}{dx} + y = \sin(2 \log(1+x))$. (7M)

Or

6. a) Solve $(D^2 + 4)y = \sec 2x$, by the method of Variation of parameters (7M)

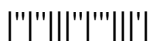
b) The charge $q(t)$ on the capacitor is given by the differential equation (7M)

$$10 \frac{d^2 q}{dt^2} + 120 \frac{dq}{dt} + 1000q = 17 \sin(2t).$$

At initial time the current is zero and the charge on the capacitor is 0.0005 coulomb. Find the charge on the capacitor for $t > 0$.

UNIT-IV

7. a) Prove that $u = \frac{x^2 - y^2}{x^2 + y^2}$, $v = \frac{2xy}{x^2 + y^2}$ are functionally dependent and find the relation between them. (7M)



b) Find the minimum value of $x^2 + y^2 + z^2$ given $x + y + z = 3a$. (7M)

Or

8. a) If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. (7M)

b) Expand $e^x \cos y$ by Taylor's theorem about the point $\left(1, \frac{\pi}{4} \right)$ up to the second degree terms. (7M)

UNIT-V

9. a) Evaluate $\iint r \, dr \, d\theta$ bounded by the inside cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$. (7M)

b) Find the area lying between the circle $x^2 + y^2 = a^2$ and the plane $x + y = a$ in the first quadrant (7M)

Or

10 a) Evaluate $\int_0^1 \int_x^{2-x} \frac{x}{y} \, dy \, dx$ by change of order of integration. (7M)

b) Using spherical polar coordinates, evaluate $\iiint xyz \, dx \, dy \, dz$ taken over the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. (7M)

