Code No: R201101 **R20** SET - 1

I B. Tech I Semester Supplementary Examinations, August/Sep-2022

MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

UNIT-I

1. a) Discuss the convergence of $\frac{x}{1.3}+\frac{x^2}{3.5}+\frac{x^3}{7.9}+....(x>0)$. (7M)

   b) Verify Lagrange's mean value theorem for $f(x) = x^3 - x^2 - 5x + 3$ in [0,4] and find the value of the constant. (7M)

Or

2. a) Examine the convergence of $\frac{3}{5}-\frac{5}{7}+\frac{7}{10}-\frac{9}{13}+....$ (7M)

   b) Find Taylor's series expansion of the $f(x, y) = \cos x$ about $x = \frac{\pi}{3}$ and hence find the approximate value of $\cos 35^\circ$. (7M)

UNIT-II

3. a) Solve $(1-x^2)\frac{dy}{dx}+xy=y^3\sin^{-1}x.$ (7M)

   b) A bacterial culture, increases from 100 to 400grams in 10 hours. How much would be present after 3 hours? (7M)

Or

4. a) Find the orthogonal trajectories of the following family of curves: $r^n = a^n \sin n\theta$. (7M)

   b) Solve $2xydy - (x^2 + y^2 + 1)dx = 0$. (7M)

UNIT-III

5. a) Solve $(D^2 + 1)\, y = \sin x \sin 2x + e^x x^2$. (7M)

   b) Solve $(x+3)^2\frac{d^2y}{dx^2}+(x+3)\frac{dy}{dx}+y=\sin(2\log(1+x))$. (7M)

Or

6. a) $Solve\ (D^2+4)y = Sec2x\ ,by\ the\ method\ of\ Variation\ of\ parameters$ (7M)

   b) The charge q(t) on the capacitor is given by the differential equation (7M)

   $10\frac{d^2q}{dt^2}+120\frac{dq}{dt}+1000q=17sin(2t).$

   At initial time the current is zero and the charge on the capacitor is 0.0005 coulomb. Find the charge on the capacitor for t>0.

UNIT-IV

7. a) Prove that $u=\frac{x^2-y^2}{x^2+y^2}$, $v=\frac{2xy}{x^2+y^2}$ are functionally dependent and find the relation between them. (7M)

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b) Find the minimum value of $x^2 + y^2 + z^2$ given x + y + z = 3a. (7M)

Or

8. a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (7M)

b) Expand $e^x \cos y$ by Taylor's theorem about the point $\left(1, \frac{\pi}{4}\right)$ up to the second degree terms. (7M)

UNIT-V

9. a) Evaluate $\iint r\, dr d\theta$ bounded by the inside cardioid $r = a(1 + cos\theta)$ and outside the circle r = a. (7M)

b) Find the area lying between the circle $x^2 + y^2 = a^2$ and the plane $x + y = a$ in the first quadrant (7M)

Or

10 a) Evaluate $\int_0^1 \int_x^{2-x} \frac{x}{y} dydx$ by change of order of integration. (7M)

b) Using spherical polar coordinates , evaluate $\iiint xyz\, dxdydz$ taken over the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. (7M)