Code No: **R201101** **R20** **SET - 1**

I B. Tech I Semester Regular/Supplementary Examinations, February - 2023

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours Max. Marks: 70

Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks

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UNIT - I

1. a) Examine the convergence of $\frac{2^{n+1}-2}{3^{n+1}+1}x^n$ for x > 0. [7M]
   b) Fine the Maclaurin series expansion of $f(x) = coshx$. [7M]

(OR)

2. a) Show that $\log(1+x)=\frac{x}{(1+\theta x)}$ where $0<\theta<1$. and hence deduce that $\frac{x}{1-x} < \ln(1+x) < x$ if x > a. [7M]
   b) Examine the convergence of $\sum_{n=0}^{\infty}(-1)^n(n+1)x^n$ with $x > \frac{1}{2}$. [7M]

UNIT - II

3. a) Solve $3y' + xy = xy^{-2}$. [7M]
   b) If a substance cools from 370k to330k in 10minutes, when the temperature of the surrounding air is 290k, find the temperature of the substance after 40 minutes. [7M]

(OR)

4. a) Show that the family of parabolas $y^2 = 4cx + 4c^2$ is "self-orthogonal". (Where *c* is a parameter). [7M]
   b) Solve $(2y^2 + 4x^2y)dx + (4xy + 3x^3)dy = 0$. [7M]

UNIT - III

5. a) Solve $(D^3 - 2D + 4)y = x^4 + 3x^2 - 5x + 2$. [7M]
   b) Determine the current *I*(*t*) in an *RLC* circuit with *emf* $E(t) = E_0 \cos \omega t$. [7M]

(OR)

6. a) Solve $(D^2 + 1)y = xcos2x$ by the method of variation of parameters. [7M]
   b) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 sin(log\, x)$. [7M]

UNIT - IV

7. a) If $w = x^2y + y^2z + z^2x$, then prove that $w_x + w_y + w_z = (x + y + z)^2$. [7M]
   b) Investigate the maxima and minima, if any, of the function$f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$. [7M]

(OR)

8. a) Determine whether the following functions are functionally dependent or not? Find a functional relation between them in case they are functionally dependent. [7M]
   $u = \frac{x+y}{x-y}, v = \frac{xy}{(x-y)^2}$.
   b) Expand $f(x, y) = e^y\, ln(1 + x)$ in powers if *x* and *y*. [7M]
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UNIT - V

9. a) Evaluate $\iint_D (1+x+y)dxdy$ where *D* is the region bounded by $y = x, x = \sqrt{y}$, *y*=1 and *y*=0. [7M]

b) Change the order of integration and thenevaluate$\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$. [7M]

(OR)

10 a) Evaluate$\int_0^2 \int_1^z \int_0^{yz} xyz\, dx\, dy\, dz$. [7M]

b) Evaluate $\int_0^{2a} \int_0^{\sqrt{2a-x^2}} dydx$ by changing into polar coordinates. [7M]

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Code No: **R201101** **R20** **SET - 2**

I B. Tech I Semester Regular/Supplementary Examinations, February - 2023

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours Max. Marks: 70

Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks

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UNIT - I

1. a) Examine the convergence of $\left[\frac{n}{n^2+1}x^{2n}\right]^{\frac{1}{2}}$. [7M]

   b) State Maclaurin's theorem with Lagrange's form of remainder for $f(x) = Cos\ x$. [7M]

(OR)

2. a) Using Lagrange's Mean Value theorem prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > cos^{-1}\frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$. [7M]

   b) Examine the convergence of $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}x^n}{n(n-1)}$ with $0 < x < 1$. [7M]

UNIT - II

3. a) Solve $2xyy' = y^2 - 2x^3$. [7M]

   b) Water at temperature 100^0C cools in 10 min to 80^0C in a room of temperature 25^0C. (i) Find the temperature of water after 20 min. [7M]
   (ii) When will the temperature be 40^0C.

(OR)

4. a) Show that the family of confocal conics $\frac{x^2}{a^2+c} + \frac{y^2}{b^2+c} = 1$ is "self-orthogonal". Here *a* and *b* are given constants. [7M]

   b) Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. [7M]

UNIT - III

5. a) Solve $(D^4 + D^3 + D^2)y = 5x^2 + sin2x + 4e^{-3x}$ [7M]

   b) Determine the current *I*(*t*) in an *RLC* circuit with *emf* $E(t) = E_0 sin\ \omega t$. [7M]

(OR)

6. a) Solve $(D^2 + 4)y = 4sec2x$ by the method of variation of parameters. [7M]

   b) Solve $x^3y''' + 2x^2y'' = x + sin(ln\ x)$. [7M]

UNIT - IV

7. a) Prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ if $u = ln(x^3 + y^3 + z^3 - 3xyz)$. [7M]

   b) Investigate the maxima and minima, if any, of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. [7M]
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(OR)

8. a) Determine whether the following functions are functionally dependent or not? Find a functional relation between them in case they are functionally dependent. [7M]
$u = \frac{x-y}{x+a}$, $v = \frac{x+a}{y+a}$ where a is constant.

b) Find Taylor's expansion of $f(x, y) = cot^{-1}xy$ in powers of $(x + 0.5)$ and $(y - 2)$ up to second degree terms. [7M]

UNIT - V

9. a) Evaluate $\iint_D (x^2 + y^2)dxdy$ where *D* is the region bounded by $y = x, y^2 = x$ and x=1 in the first quadrant. [7M]

b) Change the order of integration and then evaluate $\int_0^2 \int_{y^3}^{4\sqrt{2y}} y^2\, dx\, dy$. [7M]

(OR)

10 a) Evaluate $\int_0^a \int_0^x \int_0^{y+x} e^{x+y+z}\, dz\, dy\, dx$. [7M]

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)dxdy$ by changing into polar coordinates. [7M]

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Code No: **R201101** **R20** **SET - 3**

I B. Tech I Semester Regular/Supplementary Examinations, February - 2023
MATHEMATICS-I
(Common to All Branches)

Time: 3 hours Max. Marks: 70

Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks

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UNIT - I

1. a) Examine the convergence of $\frac{1.3.5......(2n-1)}{2.4.6.....2n}x^{n-1}$ with $x > 0$. [7M]

   b) Verify Taylor's theorem for $f(x) = x^3 - 3x^2 + 2x$ in $\left[0, \frac{1}{2}\right]$ with Lagrange's remainder up to 2 terms. [7M]

(OR)

2. a) Show that $0 < \sin b - \sin a < b - a$ if $0 < a < b < \frac{\pi}{2}$. [7M]

   b) Examine the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(x+n)}$. [7M]

UNIT - II

3. a) Solve $(xy^5 + y)dx - dy = 0$. [7M]

   b) Water at temperature 10^0C takes 5 min to warm up to 20^0C in a room at temperature 40^0C. Find the temperature after 20 min and after ½ hr. [7M]

(OR)

4. a) Show that the family of parabolas $y^2 = 2cx + c^2$ is "self-orthogonal". (where c is a parameter). [7M]

   b) Solve$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$. [7M]

UNIT - III

5. a) Solve $(D^4 + 2D^3 - 3D^2)y = 5x^2 + 7e^{2x} + 4cosx$. [7M]

   b) A circuit consists of inductance of 0.05 *henrys*, a resistance of 5 *ohms* and a condenser of capacitance 4×10^{-4}*farad*. If $Q = I = 0$ when $t = 0$, find $Q(t)$ and $I(t)$ when there is a constant *emf* of 110 *volts*. [7M]

(OR)

6. a) Solve $(D^2 + a^2)y = xcosax$ by the method of variation of parameters. [7M]

   b) Solve $x^2y'' + 5xy' + 4y = x^2 + 16\,(ln\,x)^2$. [7M]

UNIT - IV

7. a) Show that $yz_x + xz_y = x^2 - y^2$ if $e^{-\frac{z}{(x^2-y^2)}} = (x - y)$. [7M]

   b) If the total surface area of a closed rectangular box is 108 *sq. cm*, find the dimensions of the box having maximum volume. [7M]

(OR)

8. a) If $x = e^u \sec v, y = e^u \cos v$, find $J = \frac{\partial(x,y)}{\partial(u,v)}$ and $J' = \frac{\partial(u,v)}{\partial(x,y)}$. Also show that $JJ' = 1$. [7M]

   b) Expand *cos x cos y* in powers of *x* and *y* up to third degree terms. [7M]

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UNIT - V

9. a) Evaluate $\iint_D xydxdy$ where D is the domain bounded by the parabola $x^2 = 4ay$, the ordinates x=a and x-axis. [7M]

b) Change the order of integration and thene valuate $\int_0^a \int_{\frac{y^2}{a}}^{2a-y} xy\, dx\, dy$. [7M]

(OR)

10 a) Evaluate $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \int_0^{xy} cos\frac{z}{x}\, dz\, dy\, dx$. [7M]

b) Evaluate $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} xy\, dxdy$ by changing into polar coordinates. [7M]

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Code No: **R201101** **R20** **SET - 4**

I B. Tech I Semester Regular/Supplementary Examinations, February - 2023
MATHEMATICS-I
(Common to All Branches)

Time: 3 hours Max. Marks: 70

Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks
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UNIT - I

1. a) Examine the convergence of $\left(\sqrt{n^2+1}-n\right)x^{2n}$. [7M]

   b) Fine the Maclaurin's series expansion of$f(x) = sinhx$. [7M]

(OR)

2. a) If $0 \leq a < b < \frac{\pi}{2}$ then show that $0 < \cos a - \cos b < b - a$. [7M]

   b) Examine the convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2}$. [7M]

UNIT - II

3. a) Solve $y' - \cot y + xcot\ y = 0$. [7M]

   b) A body is heated to 110^0C and placed in air at10^0C. After 1 hour its temperature is 60^0C. How much additional time is required for it to cool to 30^0C? [7M]

(OR)

4. a) Show that the family of confocal conics $\frac{x^2}{a} + \frac{y^2}{a-b} = 1$ is "self-orthogonal". Here *a* is an arbitrary constant. [7M]

   b) Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$. [7M]

UNIT - III

5. a) Solve $(D^4 + 2D^3 - 3D^2)y = x^2 + 3e^{2x} + 4sinx$. [7M]

   b) A circuit consists of an inductance of 0.05 *henrys*, a resistance of 5 *ohms* and a condenser of capacitance 4×10^{-4}*farad*. If $Q = I = 0$ when $t = 0$, find $Q(t)$ and $I(t)$ when there is an alternating *emf* 200 *cos* 100*t*. [7M]

(OR)

6. a) Solve $(D^2 + 1)y = log\ cos\ x$by the method of variation of parameters. [7M]

   b) Solve $x^3y''' - 8x^2y'' + 28xy' - 40y = -\frac{9}{x}$. [7M]

UNIT - IV

7. a) If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ then show that $(u_x)^2 + \left(u_y\right)^2 + (u_z)^2 = u^4$. [7M]

   b) An aquarium with rectangular sides and bottom (and no top) is to hold 32 *liters*. Findits dimensions so that it will use the least amount of material. [7M]

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(OR)

8. a) Determine whether the following functions are functionally dependent or not? Find a functional relation between them in case they are functionally dependent. [7M]

$u = \frac{x}{y}, v = \frac{x+y}{x-y}$.

b) Obtain the expansion of e^{xy} in powers of $(x-1)$and $(y-1)$. [7M]

UNIT - V

9. a) Evaluate $\iint_D x^3 y dx dy$ where D is the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant. [7M]

b) Change the order of integration and then evaluate $\int_0^1 \int_{y^2}^{y^{1/3}} xy\, dx\, dy$. [7M]

(OR)

10 a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dx\, dy\, dz}{(x+y+z+1)^3}$. [7M]

b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. [7M]

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