Code No: **R201101** **R20** **SET - 1**

I B. Tech I Semester Supplementary Examinations, July/August-2023
MATHEMATICS-I
(Common to All Branches)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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UNIT – I

1. a) Test the Convergence of the series $\sum_{i=1}^{\infty} \tan\left(\frac{1}{n}\right)$. [7M]

   b) Test the Convergence of the series $\sum_{i=1}^{\infty}(-1)^n \frac{x^n}{1+x^n}$ [7M]

(OR)

2. a) Obtain Taylor's series Expansion of $\log(1+e^x)$ about x =0 [7M]

   b) Verify Cauchy mean value theorem for $f(x) = \frac{1}{x^2}, g(x) = \frac{1}{x}$ *on* $[a, b]$ *where* $a > b > 0$. [7M]

UNIT - II

3. a) Solve the DE $\left(x^2-1\right)\sin x\frac{dy}{dx}+\left[2x\sin x+\left(x^2-1\right)\cos x\right]y=\left(x^2-1\right)\cos x$ [7M]

   b) If the population of a country doubles in 50 years, in how many years will it triple, assuming that the rate of increase is proportional to the number of in habitants? [7M]

(OR)

4. a) Solve the DE $\frac{dy}{dx}+\left(y-1\right)\cos x = e^{-\sin x}.\cos^2 x$ [7M]

   b) Solve the D.E $y\left(xy+2x^2y^2\right)dx + x\left(xy-x^2y^2\right)dy=0$ [7M]

UNIT - III

5. a) Solve the D.E $\left(D^2+\frac{1}{x}D\right)y=\frac{12\log x}{x^2}$ [7M]

   b) Solve the D.E $\left(D^2+5D+4\right)y=x^2+7x+9$ [7M]

(OR)

6. a) Solve the D.E $\left(D^2-2D+1\right)y = xe^x\cos x$ [7M]

   b) Find the distance from the centre of which the velocity in simple harmonic motion will be (i) Half (ii) One-third of the maximum. [7M]

UNIT - IV

7. a) If $u = x\log(xy)$, where $x^3+y^3+3xy=1$. Find $\frac{du}{dx}$ [7M]

   b) Check whether the following functions are independent or dependent if so find the relation among the variable if u = xy + yz +zx , v = $x^2+y^2+z^2$ and w = x+y+z. [7M]

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(OR)

8. a) Verify Euler's theorem for the function $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{x}{y}\right)$ [7M]

b) Expand the function using Taylor's series f(x ,y) = e^x sin y about (0,0). [7M]

UNIT - V

9. a) Evaluate $\int_0^4 \int_{y^2/4}^{y} \frac{y}{x^2 + y^2}\, dx\, dy$ [7M]

b) Evaluate $\iiint_v xyz\, dx\, dy\, dz$ taken over the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant using spherical polar-coordinates. [7M]

(OR)

10 a) Evaluate $\int_{y=0}^{1} \int_{y=x}^{a} \frac{x}{x^2 + y^2}\, dx\, dy$ by changing into polar co-ordinates. [7M]

b) Evaluate $\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} dx\, dy\, dz$ By change into cylindrical co-ordinates. [7M]

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