II B. Tech I Semester Regular//Supplementary Examinations, Deceember-2023
MATHEMATICS - III
(Com to all branches)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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UNIT-I

1 a) Given $\bar{F} = (x^2 - yz)\bar{\imath} + (y^2 - xz)\bar{\jmath} + (z^2 - yx)\bar{k}$. Calculate Curl $\bar{F}$. Also Find a scalar function $\phi(x, y, z)$, such that Grad$\phi(x, y, z) = \bar{F}$. [7M]

b) Apply Stokes Theorem to evaluate $\int_c \overline{F}\,.\,d\overline{R}$, where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x + z = a$, and $\overline{F} = y\bar{\imath} + z\bar{\jmath} + x\bar{k}$. [7M]

OR

2 a) Verify Green's theorem for $\bar{F} = (y^2 + yx)\bar{\imath} + (x^2)\bar{\jmath}$, on a region R which is bounded by $y = x$ and $y = x^2$. [7M]

b) Find $curl\left(\frac{\bar{r}}{r^3}\right)$ where $\bar{r}$ represents the position vector $\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + z\,\bar{k}$. [7M]

UNIT-II

3 a) If $f(t) = \begin{cases} t, 0 \le t \le 2 \\ e^t, 2 \le t < 4 \\ Sint, t > 4 \end{cases}$, and $f(t) = t + g(t)\,H(t-2) + k(t)\,H(t-4)$ where H(t-a) is unit step function at a, then find the Laplace transform of f(t) using the result $L\big(H(t-a)f(t-a)\big) = e^{-as}L(f(t)$. [7M]

b) Solve the differential equation $(D^2 + 1)x = tCos2t$,with the conditions $x = D, x = 0$ at $t = 0$. [7M]

OR

4 a) If $L\{f(t)\} = \bar{f}(s)$, then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)ds$, provided the integral exist. Hence, evaluate $\int_0^\infty \left(\frac{\cos at - \cos bt}{t}\right) dt$. [7M]

b) $y'' + y = 2e^t, y(0) = 0, y'(0) = 2$ [7M]

UNIT-III

5 a) Using Fourier integral if $(a, b > 0)$ then show that $e^{-ax} - e^{-bx} = \frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{\lambda \sin \lambda x}{(\lambda^2+a^2)(\lambda^2+b^2)} d\lambda$,. [7M]

b) Expand $f(x) = \frac{\pi^2 - 3x^2}{12}$ as Fourier series in $(-\pi, \pi)$, $f(x+2\pi) = f(x), \forall x \in \mathcal{R}$ [7M]

OR
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6 a) Find a Fourier series to represent $f(x) = x^2, -\pi < x < \pi$, $f(x+2\pi) = f(x), \forall x \in \mathcal{R}$ Hence find the value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \infty$ [7M]

b) Find the Fourier Sine transform of $f(x) = 2e^{-5x}$ [7M]

UNIT-IV

7 a) Find the general solution of $(x^3+3xy^2)p+(y^3+3x^2y)q = 2(x^2+y^2)z$ [7M]

b) Form the Partial differential equation from $f(x + y + z, x^2 + y^2 + z^2) = 0$. [7M]

OR

8 a) Solve $p+q = \sin x + \sin y$ [7M]

b) Solve $x^2(y^2 - z^2)\frac{\partial z}{\partial x} + y^2(z^2 - x^2)\frac{\partial z}{\partial y} = z^2(x^2 - y^2)$ [7M]

UNIT-V

9 a) Solve $(D^2 - DD' - 2D)Z = \cos(3x + 2y)$ [7M]

b) Solve the boundary value problems $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, o < x < l, \frac{\partial u(0,t)}{\partial x} = 0, \frac{\partial u(l,t)}{\partial x} = 0, u(x, o) = x$ [7M]

OR

10 a) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = e^{x+y} + 2x^2y$ [7M]

b) The ends A and B of a bar of 20 cm long have the temperatures respectively at 30^0c and 80^0c until steady state prevails. If the temperatures at A and B are suddenly reduced to 0^0c and maintained at0^0c. Find the temperature in the bar at any time t. [7M]

Code No: R2021011 **R20** SET - 2

II B. Tech I Semester Regular/Supplementary Examinations, December-2023
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UNIT-I

1 a) Find the unit normal vector to the surface $x^2y + 2xz = 4$ at the point (2, -2,3). Also find the directional derivative of the surface in the direction normal to the surface $x \log z - y^2 = 1$ at (-1,2,1). [7M]

b) Verify Divergence Theorem for $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$ over the closed surface Sin the 1^{st}octant formed by the coordinate planes and the plane x+y+z = a [7M]

OR

2 a) Find the work done by $\bar{F} = (2x - y - z)\bar{i} + (x + y - z)\bar{j} + (3x - 5z - 2y)\bar{k}$ along the curve in the xy-plane given by $x^2 + y^2 = 9, z = 0$ [7M]

b) Evaluate $\int_S \text{curl}\overline{F}\,.\,\overline{n}dS$ where S is the surface of the hemisphere having center at the origin above xy-plane and $\overline{F} = (x^2 + y - 4)\overline{i} + 3xy\overline{j} + (2xz + z^2)\overline{k}$. [7M]

UNIT-II

3 a) Define unit step function u (t-a). Find the Laplace transform of u (t-a) × f (t-a). [7M]

Use this result in finding the Laplace transform of $f(t) = \begin{cases} t - 1, 1 < t \le 2 \\ 3 - t, 2 < t \le 3 \\ 0, otherwise \end{cases}$

b) Find the inverse Laplace transform of $\int_s^\infty \left[\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right] ds$ [7M]

OR

4 a) Find the Inverse Laplace Transform of $\log\left(\frac{s^2+4}{s^2+9}\right)$ [7M]

b) Using Laplace transforms, Solve the Initial value problem $y^{(2)}(t) + 2y^{(1)}(t) - y(t) = t$, given that $y(0) = 0; y^{(1)}(0) = 1$ [7M]

UNIT-III

5 a) Find the half range sine series of $f(x) = \begin{cases} Sinx \;\; for \;\; 0 \le x \le \frac{\pi}{4} \\ Cosx \;\; for \;\; \frac{\pi}{4} \le x \le \frac{\pi}{2} \end{cases}$. [7M]

b) Using Fourier Integral formula, show that $e^{-x} \cos x = \frac{2}{\pi} \int_0^\infty \frac{(\lambda^2+2) \cos \lambda x}{\lambda^4+4} d\lambda$. [7M]

OR

6 a) Find the Fourier series of $f(x) = x - x^2$ defined in $(-\pi, \pi)$, $f(x + 2\pi) = f(x) \forall x \in \mathcal{R}$ [7M]

b) Find the Fourier Sine transform of $e^{-|x|}$.Hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0$ [7M]

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UNIT-IV

7 a) Solve $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$ [7M]

b) Solve $xz\,p + yz\,q = xy$ [7M]

OR

8 a) Solve $(mz - ny)p + (nx - lz)q = (ly - mx)$. [7M]

b) Form the P.D.E by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$. [7M]

UNIT-V

9 a) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$ [7M]

b) Find all possible forms of solution of the differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. [7M]

OR

10 a) Solve by method of separation of variables, $u_x = 2u_t + u$, where $u(x, 0) = 6e^{-3x}$ [7M]

b) The temperature at one end of a bar, 50cm long with insulated sides, is kept at 0^oc and that the other end kept at 100^oc until steady state conditions prevail. The two ends are then suddenly insulated, so that the temperature gradient is zero at each end thereafter. Find temperature distribution on the bar. [7M]

Code No: R2021011 **R20** SET - 3

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UNIT-I

1 a) Evaluate the angle between the normal to the surface $xy = z^2$ at the points (4,1,2) & (3,3,-3). [7M]

b) Verify Stoke's theorem for $\overline{F} = (2x - y)\overline{i} - yz^2\overline{j} - y^2z\overline{k}$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection of the xy plane. [7M]

OR

2 a) Find for what values of n,$\frac{\overline{r}}{r^n}$ is Solenoidal and irrotational [7M]

b) Verify Greens theorem in a plane for $\int_c (x^2 + 2xy)dx + (y^2 + x^3y)dy$ where c is a square with vertices P (0,0), Q (1,0) and S (0,1). [7M]

UNIT-II

3 a) Find the inverse Laplace transform of $\frac{s}{s^2+5s+6}e^{-2s}$ [7M]

b) Sole the differential equation using Laplace transform technique $y'' + 7y' + 10y = 4e^{-3t}, y(0) = 0, y'(0) = -1$. [7M]

OR

4 a) Using Convolution Theorem Find the inverse Laplace transform of $\frac{1}{(s^2+4)(s+1)^2}$ [7M]

b) Using Laplace transform, solve$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ given that $y(0) = 0, y'(0) = 1$. [7M]

UNIT-III

5 a) Find the Fourier series of $f(x) = x^2$ defined in $(0,2)$ and f(2+x) =f(x)$\forall x \in \mathcal{R}$ [7M]

b) Find the Fourier transform of $f(x) = \begin{cases} a - |x|, for|x| < a \\ 0, for|x| > a \end{cases}$. Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ [7M]

OR

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6 a) Find the Half range sine series of $f(x) = \begin{cases} kx; 0 < x < \frac{\pi}{2} \\ k(\pi - x); \frac{\pi}{2} < x < \pi \end{cases}$ [7M]

b) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ and deduce that $\int_0 \frac{e^{-ax} - e^{-bx}}{x} \sin s\, x dx = \tan^{-1}\left(\frac{s}{a}\right) - \tan^{-1}\left(\frac{s}{b}\right)$. [7M]

UNIT-IV

7 a) Solve $(y + x)z = x^2 p + y^2 q$ [7M]

b) Form the partial differential equation by eliminating the arbitrary functions in $z = f(x + t) + g(x - t)$ [7M]

OR

8 a) Solve $(x^2 - yz)p - (y^2 - zx)q = z^2 - xy$ [7M]

b) Eliminate arbitrary function f from $f(x^2 + y^2, z - xy) = 0$ and form a differential equation. [7M]

UNIT-V

9 a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to conditions u(0, y) = u(l,y) = u(x,0)=0 and u(x, a)= $\text{Sin}\frac{\pi x}{l}$ where $0 \le x \le l$, and $0 \le y \le a$ [7M]

b) **S**olve, using method of separation of variables, the P D E $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$, given conditions are $u = 0$ and $\frac{\partial u}{\partial x} = 1 + e^{-3y}$ when x=0 for all values of y. [7M]

OR

10 a) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$. [7M]

b) A tight string of length 20 cms fastened at both ends is displaced from its position of equilibrium by imparting to each of its points an initial velocity given by $v = \begin{cases} x & in\ 0 \le x \le 10 \\ 20 - x & in\ 10 \le x \le 20 \end{cases}$; x being the distance from one end. Determine the displacement at any subsequent time. [7M]

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UNIT-I

1 a) Find Curl (Curl$\bar{R}$), where $\bar{R} = \text{grad}(x^2yz + xy^2z + xyz^2)$ [7M]

b) Evaluate $\int_S \text{curl}\bar{F}.\bar{n}dS$ where S is the surface of the hemisphere $x^2 + y^2 + z^2 = 16$ above xy-plane and $\bar{F} = (x^2 + y - 4)\bar{i} + 3xy\bar{j} + (2xz + z^2)\bar{k}$. [7M]

OR

2 a) Find the Directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point(1,1,1). [7M]

b) Verify Stoke's Theorem for $\bar{A} = (2x - y)\bar{i} - yz^2\bar{j} - y^2z\bar{k}$ over upper half of the surface of the sphere of unit radius. [7M]

UNIT-II

3 a) Define Laplace transform of a function. Find the Laplace transform of $\frac{\sin^2 t}{t}$ and hence evaluate $\int_0^\infty e^{-2t}\frac{\sin^2 t}{t}\,dt$. [7M]

b) Solve $y^{iv} - 16y = 30\sin t, y''(0) = 0, y'''(0) = -18, y''(\pi) = 0, y'''(\pi) = -18$ [7M]

OR

4 a) If $L(f(t)) = \frac{s+2}{s^2+4}$. Find the value of $\int_0^\infty f(t)dt$. [7M]

b) Solve $(D^3 - D^2 + 4D - 4)y = 68\, e^x \sin 2x$, y = 1, Dy = -19, D^2y = -37 at x = 0 using the technique of Laplace transforms. [7M]

UNIT-III

5 a) Find the Fourier series representation of the function $f(x) = x\sin x$, $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)\forall x \in \mathcal{R}$ [7M]

b) Obtain the Fourier Cosine transform of $f(x) = \frac{1}{1+x^2}$ [7M]

OR

6 a) Obtain the Fourier series of $f(x) = \frac{\pi - x^2}{4}, 0 < x < 2\pi$ and $f(x + 2\pi) = f(x)\forall x \in \mathcal{R}$ [7M]

b) Find the Fourier transformation of f(x) given by $f(x) = \begin{cases} a^2 - x^2, \text{if}|x| < a \\ 0, \quad \text{if}|x| > a \end{cases}$. Hence, prove that $\int_0^\infty \frac{\sin x - x\cos x}{x^3}dx = \frac{\pi}{4}$. [7M]
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UNIT-IV

7 a) Form the partial differential equation by eliminating arbitrary functions f and g from $z= f(x^2-y) + g(x^2+y)$ [7M]

b) $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$ [7M]

OR

8 a) Form a Partial differential Equation for which $z = F(ax + by) + G(ax - by)$ is a complete solution. [7M]

b) Solve $(y + x)z = x^2p + y^2q$ [7M]

UNIT-V

9 A bar of length 10 cms, with insulated sides has its ends A and B maintained at temperature 50 degrees and 100 degrees Celsius respectively, until steady state conditions prevail. The temperature at A is raised to 90 degrees Celsius and end B is lowered to 60 degrees Celsius. Find the distribution of temperature in the bar at any time t [14 M]

OR

10 a) Using the method of separation of variables, solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ given [7M]

$$u(0, y) = 3e^{-y} - e^{-5y}$$

b) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3 \frac{\Pi x}{l}$.Find the displacement y(x, t). [7M]