

II B. Tech I Semester Supplementary Examinations, July - 2022

MATHEMATICS - III

(Com to all branches)

Time: 3 hours

Max. Marks: 70

Answer any **FIVE** Questions each Question from each unitAll Questions carry **Equal** Marks

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- 1 a) If  $f(x, y, z) = 2x^2 + 4xy + 3z$  then find  $\text{grad } f$ . [4M]  
 b) Find the divergence of the vector function  $\vec{F} = 3x^2 \vec{i} + 5xy^2 \vec{j} + xyz^3 \vec{k}$ . [5M]  
 c) Find Curl of the vector function  $\vec{F} = e^{xyz} (i + j + k)$ . [5M]

**Or**

- 2 Verify Green's theorem in plane for  $\oint_C [(x^2 - 2xy) dx + (x^2 y + 3) dy]$ , where C [14M]

is boundary of the region defined by  $y^2 = 8x$  and  $x = 2$ .

- 3 a) Find the Laplace transform of  $2^t + \frac{\cos 2t - \cos 3t}{t} + t \sin t$ . [7M]

- b) Find  $L^{-1} \left\{ \frac{16}{(s-2)(s+2)^2} \right\}$  using convolution theorem. [7M]

**Or**

- 4 a) Find the Laplace transform of  $t e^{4t} \sin 2t$ . [4M]

- b) Use transform method to solve the differential equation [10M]

$$\frac{d^2 x}{dt^2} - 2 \frac{dx}{dt} + x = e^t \text{ with } x = 2, \frac{dx}{dt} = -1 \text{ at } t = 0.$$

- 5 a) Find the Fourier series for the function  $f(x) = e^{ax}$  in  $(-\pi, \pi)$ . [7M]

- b) Express  $f(x) = x$  as a half-range cosine series in  $0 < x < 2$ . [7M]

**Or**

- 6 a) Find the Fourier cosine integral and Fourier sine integral of [7M]  
 $f(x) = e^{-ax} - e^{-bx}$ ,  $a > 0, b > 0$ .

- b) Find the Fourier cosine transform of  $f(x) = e^{-ax} \cos ax$ . [7M]

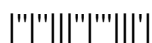
- 7 a) Form the partial differential equation by elimination the arbitrary function f [5M]

$$\text{from the relation } z = x^2 + 2f\left(\frac{1}{y} + \log x\right).$$

- b) Solve the partial differential equation [5M]

$$x^2 (y - z) p + y^2 (z - x) q = z^2 (x - y).$$

- c) Find the complete integral of  $pe^y = qe^x$ . [4M]

**Or**

- 8 a) Form PDE from  $z = ax + by + a^2 + b^2$ . [4M]  
b) Solve the partial differential equation [5M]  
 $\tan x p + \tan y q = \tan z$ .  
c) Find the complete integral of  $p^3 - q^3 = 0$ . [5M]
- 9 a) Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$ . [7M]  
b) Solve  $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + 4 \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$ . [7M]

**Or**

- 10 Solve the wave equation  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$  under the conditions [14M]  
 $y(0,t) = 0$ ,  $y(L,t) = 0$  for all  $t$ ;  
 $y(x,0) = f(x)$  and  $\left(\frac{\partial y}{\partial t}\right)_{t=0} = g(x)$ ,  $0 < x < L$ .

