Code No: R2021011 **R20** SET - 1

II B. Tech I Semester Regular/Supplementary Examinations, January - 2023
MATHEMATICS - III
(Com to all branches, Except EEE &FE)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions, each Question from each unit
All Questions carry **Equal** Marks
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UNIT-I

1 a) Apply Stoke's theorem, to evaluate $\oint_c (ydx + zdy + xdz)$ where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. [7M]

b) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential. [7M]

OR

2 a) Evaluate $\iint_s \bar{F}.\bar{n}\, ds$ where $\bar{F} = 12x^2y\bar{i} - 3yz\bar{j} + 2z\bar{k}$ and S is the portion of the plane $x + y + z = 1$ included in the first octant. [7M]

b) Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$. [7M]

UNIT-II

3 a) Solve the differential equation $\frac{d^2x}{dt^2} + 9x = \sin t$ using Laplace Transforms given that $x(0) = 1, x'(0) = a, x(\pi/2) = 1$. [9M]

b) Find the inverse Laplace transform of $\frac{2+5s}{s^2e^{4s}}$ [5M]

OR

4 a) Solve using Laplace transforms $y^{(iv)} - 16y = 30\sin t$, given that $y(0) = 0$, $y'(0) = -18, y''(\pi) = 0, y'''(\pi) = -18$ [7M]

b) Find $L\left\{\int_0^t \frac{1-e^{-u}}{u}\,du\right\}$ [7M]

UNIT-III

5 a) Obtain the Fourier series for the function $f(x) = \begin{cases} x, 0 \le x \le \pi \\ 2\pi - x, \pi \le x \le 2\pi \end{cases}$ [7M]

and show that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

b) Using Fourier integral show that [7M]

$$e^{-ax} - e^{-bx} = \frac{2(b^2 - a^2)}{\pi}\int_0^\infty \frac{\lambda \sin \lambda x}{(\lambda^2 + a^2)(\lambda^2 + b^2)}\,d\lambda, \quad (a, b > 0).$$

OR

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6 a) Find the Fourier series of the function $f(x) = \begin{cases} x & for -1 < x < 0 \\ x+2 & for \; 0 < x < 1 \end{cases}$. [7M]
And hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$

b) Express $f(x) = \begin{cases} 1 & for \; 0 \le x \le \pi \\ 0 & for \; x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate [7M]
$\int_0^\infty \frac{1-\cos(\pi\lambda)}{\lambda} \sin x\lambda \, d\lambda$.

UNIT-IV

7 a) Form the Partial differential equation from $f(xyz, x+y) = 0$ by elimination of arbitrary function. [7M]

b) Solve $z^2(p^2 + q^2) = x^2 + y^2$. [7M]

OR

8 a) Form the Partial differential equation from $f(x+y+z, x^2+y^2+z^2) = 0$ by elimination of arbitrary function [7M]

b) Solve $(xz)p - (yz)q = (y^2 - x^2)$ [7M]

UNIT-V

9 a) Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial z^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\,sin(3x + 2y)$ [7M]

b) By the method of separation of variables, find the solution of the P.D.E [7M]
$2\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = 3u, u(x,0) = 4e^{-x}$.

OR

10 a) Solve, using method of separation of variables, the P D E $\frac{\partial u}{\partial y} + 2u = \frac{\partial^2 u}{\partial x^2}$, [7M]
given conditions are $u = 0 \; and \; \frac{\partial u}{\partial x} = 1 + e^{-3y}$ when x=0 for all values of y.

b) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = sin(2x + 2y)$ [7M]

Code No: R2021011 **R20** SET - 2

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Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
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UNIT-I

1 a) State Stoke's theorem and Verify Stoke's theorem for $\overline{F} = (2x - y)\overline{i} - yz^2\overline{j} - y^2z\overline{k}$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection of the xy plane. [7M]

b) Find the work done in moving a particle in the force field $\overline{F} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$ along the straight line from (0,0,0) to (2,1,3). [7M]

OR

2 a) Evaluate by using Green's theorem for $\int_c [(xy + y^2)dx + x^2dy]$, where C is bounded by $y = x$ and $y = x^2$. [7M]

b) Prove that $\nabla\left[\nabla.\frac{\overline{r}}{r}\right] = \frac{-2}{r^3}\overline{r}$ where $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$. [7M]

UNIT-II

3 a) Solve using the Laplace transform technique [7M]

$$y'' + y = 2e^t, y(0) = 0, y'(0) = 2$$

b) Evaluate $L\left\{t\int_0^t e^{-u} \sin 2u du\right\}$ [7M]

OR

4 a) Using Laplace transform solve $(D^2 + 3D + 2)y = 3, y(0) = y'(0) = 1$ [7M]

b) Using Laplace transform evaluate $\int_0^\infty \frac{e^{-at}\sin^2 t}{t}dt$. [7M]

UNIT-III

5 a) Is the function defined as $f(x) = \begin{cases} x+\pi, & 0 \le x \le \pi \\ x-\pi, & -\pi < x \le 0 \end{cases}$ even or odd? If $f(x+2\pi) = f(x)$, find its Fourier series expansion. [7M]

b) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & if\ |x| \le 1 \\ 0, & if\ |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\frac{x}{2}dx$ [7M]

OR

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6 a) Find the half range cosine series of $f(x) = x(2-x)\ in\ \ 0 \le x \le 2$ [7M]

b) Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}, -\infty < x < \infty$ [7M]

UNIT-IV

7 a) Solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x-y)$. [7M]

b) Form partial differential equation by eliminating the arbitrary function $f(x)$ and $g(x)$ from $z = yf(x) + xg(y)$. [7M]

OR

8 a) Solve $q^2y^2 = z(z - px)$. [7M]

b) Form the Partial differential equation from $f(x+y+z, x^2+y^2+z^2) = 0$. [7M]

UNIT-V

9 a) Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ corresponding to the triangular initial deflection $f(x) = \begin{cases} \frac{2k}{l}x, 0 < x < \left(\frac{l}{2}\right) \\ \frac{2k}{l}(l-x), \left(\frac{l}{2}\right) < x < l \end{cases}$, and zero initial velocity. [14 M]

OR

10 a) Solve the partial differential equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, which satisfies the conditions, $u(0,t) = 0$,$u(l,t) = 0$ for $t > 0$ $u(x,0) = \begin{cases} x, 0 < x < \frac{l}{2} \\ l - x, \frac{l}{2} < x < l \end{cases}$ [14M]

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UNIT-I

1 a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P = (1,2,3)$ in the direction of the line PQ where $Q = (5,0,4)$. [7M]

b) By transforming into triple integral, evaluate $\iint x^3 dydz + x^2 ydzdx + x^2 zdxdy$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs $z = 0, z = b$. [7M]

OR

2 a) Evaluate $\int \overline{F}.\overline{n}ds$ where $\overline{F} = z\overline{i} + x\overline{j} - 3y^2 z\overline{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between z=0 and z=5 [7M]

b) State Green's theorem. Evaluate by Green's theorem $\oint_c (y - \sin x)dx + \cos x\, dy$ where C is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}, \pi y = 2x$. [7M]

UNIT-II

3 a) Use convolution theorem to find inverse LaPlace transform of $\frac{1}{(s^2+4)(s+1)^2}$ [7M]

b) Find the inverse Laplace transform of $\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}$ [7M]

OR

4 a) Find the Laplace transform of $f(t) = \begin{cases} \sin t\,, t > \pi \\ \cos t\,, t < \pi \end{cases}$ [7M]

b) Solve $y'' - 8y' + 15y = 9te^{2t}, y(0) = 5, y'(0) = 10$ using the Laplace transform technique. [7M]

UNIT-III

5 a) Find the Fourier series to represent the function $f(x) = x \sin x\,, -\pi < x < \pi$. Hence deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \ldots\ldots = \frac{1}{4}(\pi - 2)$ [7M]

b) Find the Fourier sin integral of $f(x) = \begin{cases} x, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ [7M]

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6 a) If $f(x) = \begin{cases} \pi x, 0 < x < 1 \\ \pi(2-x),\ 1 < x < 2 \end{cases}$ and f(x+2) =f(x) for all x. Obtain Fourier series of f(x) . [7M]

b) Solve for $f(x)$ the integral equation $\int_0^\infty f(x)\sin xt\,dx = \begin{cases} 1, & 0 \le t < 1 \\ 2, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$ [7M]

UNIT-IV

7 a) Solve $(mz - ny)p + (nx - lz)q = (ly - mx)$. [7M]

b) Obtain partial differential equation from $z = f(2x + y) + g(3x - y)$. [7M]

OR

8 a) Solve$\big((x^2)(y^2 - z^2)\big)p + \big((y^2)(z^2 - x^2)\big)q = \big((z^2)(x^2 - y^2)\big)$. [7M]

b) Form the partial differential equation by eliminating the arbitrary constants a and b from $z = ax + by + \left(\frac{a}{b}\right) - b$. [7M]

UNIT-V

9 a) Solve by method of separation of variables the partial differential equation $u_x = 2u_t + u$, where $u(x,0) = 6e^{-3x}$ [7M]

b) The ends A and B of a bar 20 cm long have the temperatures 300°Cand 80°C until steady prevails. If the temperatures at A and B are suddenly reduced to 0^oc and maintained at 0°C. Find the temperature in a bar. [7M]

OR

10 a) Solve by Variables separable method, find all possible solutions of [7M]

$$\frac{\partial^2 z}{\partial u^2} - 2\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0$$

b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$. [7M]

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UNIT-I

1 a) Verify Divergence theorem for $\overline{F} = 2x^2yi - y^2j + 4xz^2k$ taken over the region of first octant of the cylinder $y^2 + z^2 = 9$ and $x = 0$, $x = 2$. [7M]

b) Find the angle between the normals to the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3). [7M]

OR

2 a) Find the values of a and b so that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at the point (1,-1,2). [7M]

b) Find for what values of n, $\frac{\overline{r}}{r^n}$ is solenoidal and irrotational. [7M]

UNIT-II

3 a) Find the Laplace transforms of $(\sin t - \cos t)^3$ [7M]

b) Solve the Initial value problem $\frac{d^2x}{dt^2} + 9x = \sin t\,, x(0) = 1, x\left(\frac{\pi}{2}\right) = 1$ [7M]

OR

4 a) Evaluate $\int_0^\infty t^3e^{-t} \sin t\ dt$ using the Laplace transforms. [7M]

b) Solve the Initial value problem $y'' + n^2\,y = a\,\sin(\,nt + \theta), y(0) = y'(0) = 0$ [7M]

UNIT-III

5 a) Find the Fourier series of $f(t) = \begin{cases} 1 + t^2, 0 \le t \le 1 \\ 3 - t, 1 \le t \le 2 \end{cases}$, f(t+2)=f(t) for all t [7M]

b) Find the finite Fourier sine transform and finite Fourier cosine transform of $f(x) = 2x$ in $0 < x < 4$ [7M]

OR

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6 a) Find the half range sine series of $f(x) = \begin{cases} Sinx \ \ for 0 \leq x \leq \frac{\pi}{2} \\ Cosx \ \ for \frac{\pi}{2} \leq x \leq \pi \end{cases}$ [7M]

b) Using Fourier integral representation, show that [7M]

$$\int_0^\infty \frac{\sin s \cdot \cos xs}{s} ds = \begin{cases} \frac{\pi}{2}, \ \ if \ \ 0 \leq x < 1 \\ \frac{\pi}{4}, \ \ if \ \ x = 1 \\ 0, \ \ if \ \ x > 1 \end{cases}$$

UNIT-IV

7 a) Find the general solution of $(x^3+3xy^2)p+(y^3+3x^2y)q = 2(x^2+y^2)z$ [7M]

b) Form the P.D.E by eliminating the arbitrary constants from [7M]

$$z = (x^2 + a)(y^2 + b).$$

OR

8 a) Solve $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$ [7M]

b) Solve $p^2 + q^2 = npq$. [7M]

UNIT-V

9 a) Solve by the method of separation of variables for all forms of solution to [7M]

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

b) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t [7M]

OR

10 a) A tightly stretched string of length L is fixed at its both the ends. The midpoint of the rod is taken to a height of H and then released from rest in that position. Find the displacement of any point of the string at a position x measured from one end of the rod and at any time t. [7M]

b) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$ [7M]