

**II B. Tech I Semester Supplementary Examinations, July - 2023**  
**MATHEMATICS - III**  
 (Com to all branches)

Time: 3 hours

Max. Marks: 70

Answer any **FIVE** Questions, each Question from each unit  
 All Questions carry **Equal** Marks

UNIT-I

- 1 a) If  $a = x + y + z, b = x^2 + y^2 + z^2, c = xy + yz + zx$  prove that [7M]  
 $[\text{grad } a \text{ grad } b \text{ grad } c] = 0$
- b) Find work done in moving particle in the force field  $\vec{F} = 3x^2 \vec{i} + (2xz - y) \vec{j} + z \vec{k}$  [7M]  
 along the space curve  $x = 2t^3, y = t, z = 4t^2 - t$  from  $t = 0$  to  $t = 1$ .

OR

- 2 a) Show that the vector  $(x^2 - yz) \vec{i} + (y^2 - zx) \vec{j} + (z^2 - xy) \vec{k}$  is irrotational and find [7M]  
 its scalar potential
- b) Evaluate  $\iint_s (\text{Curl } \vec{A} \cdot \vec{n}) ds$  where  $\vec{A} = y \vec{i} + (x - 2z) \vec{j} - xy \vec{k}$  and  $s$  is the surface of the [7M]  
 sphere  $x^2 + y^2 + z^2 = 4$  above  $xy$  plane by stoke's theorem.

UNIT-II

- 3 a) If  $L\{f(t)\} = \log\left(\frac{s+3}{s+1}\right)$  then find  $L\{f(2t)\}$  using change of scale property [7M]
- b) Find  $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$  [7M]

OR

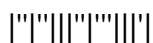
- 4 a) Find Laplace transform of  $e^{-3t}(\cos 4t + 3\sin 4t)$  [7M]
- b) Find inverse Laplace transform of  $\frac{s+5}{(s-1)^2(s+2)}$  [7M]

UNIT-III

- 5 a) Find the Fourier series of  $f(x) = \frac{1}{4}(\pi - x)^2, 0 < x < 2\pi$  [7M]  
 Hence deduce that  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$
- b) Find the Half range cosine series of  $f(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & 1 < x < 2 \end{cases}$  in  $[0, 2]$  [7M]

OR

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- 6 a) Find the Fourier transform of  $f(x)$  defined by  $f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 1-x & \text{if } 1 < x < 2 \\ 0 & \text{if } x > 2 \end{cases}$  [7M]
- b) Find the Fourier cosine transform of  $\frac{1}{1+x^2}$  [7M]

## UNIT-IV

- 7 a) Find the partial differential equation by eliminating arbitrary function from  $z = f(x^2 - y) + g(x^2 + y)$  [7M]
- b) Solve the P.D.E  $p^2 q^3 = 1$  [7M]

## OR

- 8 a) Find the partial differential equation by eliminating arbitrary constants from  $x^2 + y^2 = (z - c)^2 \cot^2 \alpha$  [7M]
- b) Solve the P.D.E  $(x + 2z)p + (4z - y)q = 2x + y$  [7M]

## UNIT-V

- 9 Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  subject to the following conditions [14M]
- (i)  $u(0, y) = 0$  for all  $y$
- (ii)  $u(a, y) = 0$  for all  $y$
- (iii)  $u(x, \infty) = 0, 0 \leq x \leq a$
- (iv)  $u(x, 0) = kx, 0 \leq x \leq a$

## OR

- 10 a) Solve the P.D.E  $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$  using method of separation of variables [7M]
- b) Solve the P.D.E  $(D^3 - 4D^2D^1 + 5DD^1^2 - 2D^1^3)z = e^{2x+y} + e^{x+y}$  [7M]

