Code No: R2021011 **R20** SET - 1

II B. Tech I Semester Supplementary Examinations, July - 2023
MATHEMATICS - III
(Com to all branches)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions, each Question from each unit
All Questions carry **Equal** Marks
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UNIT-I

1 a) If $a = x + y + z, b = x^2 + y^2 + z^2, c = xy + yz + zx$ prove that [grad a grad b grad c] =0 [7M]

b) Find work done in moving particle in the force field $\bar{F} = 3x^2\,\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the space curve $x = 2t^3$, $y = t$, $z = 4t^2 - t$ from $t = 0$ to $t = 1$. [7M]

OR

2 a) Show that the vector $\left(x^2 - yz\right)\bar{i} + \left(y^2 - zx\right)\bar{j} + \left(z^2 - xy\right)\bar{k}$ is irrotational and find its scalar potential [7M]

b) Evaluate $\iint_s (\text{Curl } \bar{A}.\bar{n})ds$ where $\bar{A} = y\,\bar{i} + (x - 2z)\bar{j} - xy\,\bar{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = 4$ above xy plane by stoke's theorem. [7M]

UNIT-II

3 a) If $L\{f(t)\} = log\left(\frac{s+3}{s+1}\right)$ then find $L\{f(2t)\}$ using change of scale property [7M]

b) Find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$ [7M]

OR

4 a) Find Laplace transform of $e^{-3t}(cos4t + 3sin4t)$ [7M]

b) Find inverse Laplace transform of $\frac{s+5}{(s-1)^2\,(s+2)}$ [7M]

UNIT-III

5 a) Find the Fourier series of f(x)= $\frac{1}{4}(\pi - x)^2$, $0<x<2\pi$

Hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$ [7M]

b) Find the Half range cosine series of $f(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & 1 < x < 2 \end{cases}$ in [0,2] [7M]

OR
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6 a) Find the Fourier transform of $f(x)$ defend by $f(x)=\begin{cases} x & if\ 0<x<1 \\ 1-x & if\ 1<x<2 \\ 0 & if\ x>2 \end{cases}$ [7M]

b) Find the Fourier cosine transform of $\frac{1}{1+x^2}$ [7M]

UNIT-IV

7 a) Find the partial differential equation by eliminating arbitrary function from $z=f(x^2-y)+g(x^2+y)$ [7M]

b) Solve the P.D.E $p^2q^3=1$ [7M]

OR

8 a) Find the partial differential equation by eliminating arbitrary constants from $x^2+y^2=(z-c)^2\cot^2\alpha$ [7M]

b) Solve the P.D.E $(x+2z)p+(4z-y)q=2x+y$ [7M]

UNIT-V

9 Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ subject to the following conditions [14M]

$(i)\ u(0,y)=0\ for\ all\ y$

$(ii)\ u(a,y)=0\ for\ all\ y$

$(iii)\ u(x,\infty)=0\ ,0\le x\le a$

$(iv)\ u(x,0)=kx,\ 0\le x\le a$

OR

10 a) Solve the P.D.E $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ using method of separation of variables [7M]

b) Solve the P.D.E $\left(D^3-4D^2D^1+5DD^{1^2}-2D^{1^3}\right)z=e^{2x+y}+e^{x+y}$ [7M]