Code No: R2021021 **R20** SET - 1

II B. Tech I Semester Regular/Supplementary Examinations, January - 2023
MATHEMATICS-IV
(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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UNIT-I

1 a) If $f(z)$ is an analytic function in a domain D and $|f(z)| = k =$ constant, show that $f(z)$is constant. [7M]

b) Evaluate, using Cauchy's integral formula $\oint_C \frac{\cos \pi z}{z^2-1}\, dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$. [7M]

OR

2 a) Prove that the function $f(z)$ defined by $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ $(z \neq 0)$, $f(0) = 0$ is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist. [7M]

b) Evaluate $\oint_C \frac{e^z}{(z^2+\pi^2)^2} dz$, where C is $|z| = 4$. [7M]

UNIT-II

3 a) Expand $f(z) = \frac{1}{[(z-1)(z-2)]}$ in the region: [7M]
(a) $|z| < 1$, (b) $1 < |z| < 2$, (c)$|z| > 2$, (d) $0 < |z-1| < 1$.

b) Evaluate $\oint_C \frac{z-3}{z^2+2z+5}\, dz$, where C is the circle (i) $|z| = 1$, (ii)$|z+1-i| = 2$, (iii)$|z+1+i| = 2$. [7M]

OR

4 a) Find the nature and location of singularities of the following functions: [7M]
(i) $\frac{z-\sin z}{z^2}$ (ii)$(z+1)\sin\frac{1}{z-2}$ (iii) $\frac{1}{\cos z-\sin z}$.

b) Evaluate $\int_0^\infty \frac{\sin mx}{x} dx$, when $m > 0$. [7M]

UNIT-III

5 a) X is a continuous random variable with probability density function given by [7M]

f(x)= kx (0≤x<2)
= 2k (2≤x<4)
=-kx+6k (4≤x<6).
Find k and mean value of X.

b) The probability density p(x) of a continuous random variable is given by p(x)=$y_0 e^{-|x|}$, -∞<x<∞. Prove that y_0=1/2. Find the mean and variance of the distribution. [7M]

OR
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6 a) Fit a binomial distribution for the following data and compare the theoretical frequencies with the actual once: [7M]

x :	0	1	2	3	4	5
f:	2	14	20	34	22	8

b) X is a normal variate with mean 30 and S.D. 5, find the probabilities that (i) $26 \leq X \leq 40$, (ii) $X \geq 45$ and (iii) $|X-30|>5$. [7M]

UNIT-IV

7 a) If X and Y are two independent χ^2 variates with ν_1 and ν_2 d.f. respectively, then find the distribution for (i) U=X+Y (ii) $V = \frac{X}{Y}$ [7M]

b) A die was thrown 60 times and the following frequency distribution was observed: [7M]

Faces :	1	2	3	4	5	6
f_0 :	15	6	4	7	11	17

Test whether the die is unbiased?

OR

8 a) The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53,51. Does the mean of these differ significantly from the assumed mean of 47.5? [7M]

b) Ten individuals were chosen at random from a normal population and their heights were found to be in inches as 63, 63,66,67,68,68,69,70,71 and 71. Test the hypothesis that the mean height of the population is 66inches. Also, find the 95% confidence limits for the true population mean μ. [7M]

UNIT-V

9 a) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. [7M]

b) Samples of two types of electric light bulbs were tested for length of life and following data were obtained [7M]

	Type I	Type II
Sample sizes :	n_1=8	n_2=7
Sample means:	$\bar{x}_1 = 1324\ hrs$	$\bar{x}_2 = 1036\ hrs$
Sample S.D. :	$s_1 = 36\ hrs$	$s_2 = 40\ hrs$

Does the data support the hypothesis that Type I is superior to Type II regarding length of life?

OR

10 a) Playing 10 rounds of golf on his home course, a golf professional averaged 71.3 with a S.D. 1.32. Test the null hypothesis at $\alpha = 0.05$ that consistency of his game on his home course is actually measured by $\sigma = 1.20$ against the alternative that he is less consistent. [7M]

b) If a random sample of 120 tractors produced by a company 47 are defective is the claim by the company that at most 30% of the tractors are defective use 0.05 level of significance. [7M]

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(Electrical and Electronics Engineering)

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UNIT-I

1 a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though CR equations are satisfied [7M]

b) Evaluate $\oint_C \frac{\sin^2 z}{\left(z-\frac{\pi}{6}\right)^3} dz$, where C is the circle $|z| = 1$. [7M]

OR

2 a) Determine the analytic function $f(z) = u + iv$, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f\left(\frac{\pi}{2}\right) = 0$. [7M]

b) Evaluate $\int_C \frac{z^2-z+1}{z-1} dz$, where C is the circle (i) $|z| = 1$ (ii)$|z| = \frac{1}{2}$. [7M]

UNIT-II

3 a) Find the Laurents' expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z + 1 < 3$. [7M]

b) Evaluate $\oint_C \frac{e^z}{\cos \pi z} dz$, where C is the unit circle $|z| = 1$. [7M]

OR

4 a) Find the sum of the residues of $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$ [7M]

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ [7M]

UNIT-III

5 a) The probability density function of a variate X is [7M]

X:	0	1	2	3	4	5	6
P(X):	k	3k	5k	7k	9k	11k	13k

(i) Find P(X<4), P(X≥5), P(3<X≤6).
(ii) What will be the minimum value of k so that P(X≤2)>3.

b) A function is defined as under : [7M]

f(x)= 1/k, $x_1 \le x \le x_2$
=0, elsewhere.

Find the cumulative distribution of the variate x when k satisfies the requirements for f(x) to be a density function.

OR

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6 a) The following data are the number of seeds germinating out of 10 on damp filter paper for 80 sets of seeds. Fit a binomial distribution to these data : [7M]

x :	0	1	2	3	4	5	6	7	8	9	10
f:	6	20	28	12	8	6	0	0	0	0	0

b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. [7M]

UNIT-IV

7 a) Prove that $(n-1)S^2/\sigma^2$ is distributed like a χ^2 variate with (n-1) d.f., where $(n-1)S^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$. [7M]

b) Five dice were thrown 96 times and the number of times 4, 5 or 6 were thrown were: [7M]

No. of dice showing 4, 5 or 6:	5	4	3	2	1	0
Frequency :	8	18	35	24	10	1

Find the probability of getting this result by chance?

OR

8 a) Find the student's t for the following variable values in a sample of eight: -4, -2, -2, 0, 2, 2, 3, 3 ; taking the mean of the universe to be zero. [7M]

b) The mean weekly sales of TVs of a particular brand in company's showrooms was 14.6 TV per showroom. After announcing a few incentives the mean weekly sales in 22 stores for a typical week increased to 15.4 with S.D. of 1.7. Were the incentives announced effective in boosting the sale? [7M]

UNIT-V

9 a) A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate an unbiased die? [7M]

b) The following random samples are measurements of the heat producing capacity in millions of calories per ton of specimens of coal from two mines: [7M]

Mine I: 8,260 8,130 8,350 8,070, 8,340
MineII: 7,950 7,890 7,900 8,140 7,920 7,840.
Test at 5% level of significance whether the difference between the means of these two samples is significant.

OR

10 a) The following figures show the distribution of digits in numbers chosen at random from a telephone directory: [7M]

Digits :	0	1	2	3	4	5	6	7	8	9	total
Frequency:	1026	1107	997	966	1075	933	1107	972	964	853	10,000

Test the hypothesis that digits occur with equal frequency in the directory.

b) In a sample of 90 university professors 28 own computers. Can we conclude at 0.05 level of significant that at most ¼ of the professors own computer? [7M]

Code No: R2021021 **R20** SET - 3

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(Electrical and Electronics Engineering)

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Answer any **FIVE** Questions, each Question from each unit
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UNIT-I

1 a) Show that polar form of Cauchy-Riemann equations are $\frac{\partial U}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$. [7M]
Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

b) Verify Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with the vertices at points $1+i, -1+i$ and $-1-i$. [7M]

OR

2 a) Find the analytic function $z = u + iv$, if $u + v = \frac{2\sin 2x}{e^{2y} - e^{-2y} - 2\cos 2x}$. [7M]

b) Evaluate, using Cauchy's integral formulae $\oint_C \frac{\log z}{(z-1)^3} dz$, where C is $|z-1| = \frac{1}{2}$. [7M]

UNIT-II

3 a) Find Taylor's expansion of (i) $f(z) = \frac{1}{(z+1)^2}$ about the point $z = -i$. [7M]
(ii) $f(z) = \frac{2z^3+1}{z^2+z}$ about the point $z = i$.

b) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. [7M]

OR

4 a) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. Hence evaluate $\oint_c f(z)dz$, where C is the circle $|z| = 2.5$. [7M]

b) By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$. [7M]

UNIT-III

5 a) A random variable X has the following probability function : [7M]

X :	0	1	2	3	4	5	6	7
P(X):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find the value of the k (ii) Evaluate P(X<6), P(X≥6)
(iii) P(0< X<5).

b) A variable X has the probability distribution [7M]

x:	-3	6	9
P(X=x):	1/6	1/2	1/3

Find E(X) and $E(X^2)$. Hence evaluate $E(2X+1)^2$.

OR

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Code No: R2021021 **R20** SET - 3

6 a) Fit a Poisson distribution to the set of observations : [7M]

x:	0	1	2	3	4
f:	122	60	15	2	1

b) The mean height of 500 students is 151cm. and the standard deviation is 15cm. Assuming that the heights are normally distributed, find how many students heights lie between 120 and 155cm. [7M]

UNIT-IV

7 a) Find the m.g.f. of a standard χ^2 with variate and obtain its limiting form as $v \to \infty$. [7M]

b) Fit a normal distribution to the following data of weights of 100 students of Delhi University and test the goodness of fit. [7M]

Weights (kg) :	60-62	63-65	66-68	69-71	72-74
Frequency:	5	18	42	27	8

OR

8 a) A sample of 10 measurements of the diameter of a sphere gave a mean of 12cm and a standard deviation 0.15cm. Find 95% confidence limits for the actual diameter. [7M]

b) The specifications for a certain kind of ribbon call for a mean breaking strength of 180 pounds. If five randomly selected pieces of the ribbon have a mean breaking strength of 169.5 pounds with a S.D. of 5.7 pound, test the null hypothesis $\mu = 180$ against the alternative hypothesis $\mu < 180$ at 5% level of significance. [7M]

UNIT-V

9 a) In a city A 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? [7M]

b) To test the claim that the resistance of electric wire can be reduced by at least 0.05 ohm alloying, 25 measurements obtained for each alloyed wire and standard wire produced the following results: [7M]

	Mean	S.D.
Alloyed wire (x)	0.083 ohm	0.003 ohm
Standard wire (y)	0.136 ohm	0.002ohm

Test the claim at 5% level of significance.

OR

10 a) A survey of 800 families with four children each recorded the following distribution: [7M]

No. of boys	: 0	1	2	3	4
No. of girls	: 4	3	2	1	0
No. of families	: 32	178	290	236	64

Test the hypothesis that male and female births are equally likely.

b) A hospital claims that at least 40% of the patients admitted are for emergency ward. Is there reason to believe this claim if the records shows that only 49 of 150 patients are for emergency ward . [7M]

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UNIT-I

1 a) Find the conjugate harmonic of $v(r,\theta) = r^2\cos 2\theta - r\cos\theta + 2$. Show that v is harmonic. [7M]

b) If $F(\zeta) = \oint_C \frac{4z^2+z+5}{z-\zeta}dz$, where C is the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$, find the value of (a) $F(3.5)$; (b)$F(i), F''(-1)$ and $F''(-i)$. [7M]

OR

2 a) Show that $u(x,y)$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ when $u(x,y) = 2x - x^3 + 3xy^2$. [7M]

b) Use Cauchy's integral formula to calculate $\oint_C \frac{\sin\pi z+\cos\pi z}{(z-1)(z-2)}dz$ where C is $|z| = 4$. [7M]

UNIT-II

3 a) Find the Laurent's expansion of $\frac{1}{[(z^2+1)(z^2+2)]}$ for (a) $0 < |z| < 1$; (b) $1 < |z| < \sqrt{2}$; (c) $|z| > 2$. [7M]

b) Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its poles and hence evaluate $\oint_C f(z)\,dz$ where C is the circle $|z| = 2.5$. [7M]

OR

4 a) Evaluate $\oint_C \frac{z\,dz}{(z-1)(z-2)^2}$, C:$|z-2| = \frac{1}{2}$. [7M]

b) Show that $\int_0^{2\pi} \frac{\cos 2\theta\, d\theta}{1-2a\cos\theta+a^2} = \frac{2\pi a^2}{1-a^2}, (a^2 < 1)$. [7M]

UNIT-III

5 a) A die is tossed thrice. A success is 'getting 1 or 6' on a toss. Find the mean and variance of the number of successes. [7M]

b) If $f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 2 & elsewhere \end{cases}$, [7M]
Represents the density of a random variable X, find E(X) and Var (X).

OR

6 a) Fit a Poisson distribution to the following : [7M]

x:	0	1	2	3	4
f:	46	38	22	9	1

b) For a normally distributed vairate with mean 1 and S.D. 3, find the probabilities that (i) $3.43 \le x \le 6.19$ (ii) $-1.43 \le x \le 6.19$. [7M]

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Code No: R2021021 **R20** SET - 4

UNIT-IV

7 a) If the sum of two independent positive variables is a χ^2 variate with (m+n) d.f., and if one of themis a χ^2 variate with m d.f., then show that other is a χ^2 variate n d.f. [7M]

b) Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05. [7M]

x :	0	1	2	3	4
f :	419	352	154	56	19

OR

8 a) A machine is supposed to produce washers of mean thickness 0.12cm. A sample of 10 washers was found to have a mean thickness of 0.128cm and standard deviation 0.008. Test whether the machine is working in proper order at 5% level of significance. [7M]

b) Construct 99% confidence interval for the true mean weight loss if 16 persons on diet control after one month had a mean weight loss of 3.42kg with S.D of 0.68 kg [7M]

UNIT-V

9 a) In a locality containing 18000 families, a sample of 840 families was selected at random. Of these 840 families, 206 families were found to have a monthly income of ₹250 or less. It is desired to estimate how many out of 18,000 families have a monthly income of ₹250 or less. Within what limits would you place your estimate? [7M]

b) The yields of two types Type I and Type II of grains in pounds per acre in 6 replications are given below. Give your comments on the difference in the mean yields. [7M]

Replication	Type I	Type II
1	205	248
2	246	263
3	230	282
4	300	308
5	304	300
6	238	220

OR

10 a) The following data give the life of 40 similar car batteries recorded to the nearest length of years [7M]

Class :	1.5-1.9	2.0-2.4	2.5-2.9	3.0-3.4	3.5-3.9	4.0-4.4	4.5-4.9
Frequency:	2	1	4	15	10	5	3

Test the hypothesis that the frequency distribution of battery lives may be approximated by a normal distribution with mean $\mu = 3.5$ and S.D. $\sigma = 0.7$.

b) It is observed that 174 out of a random sample of 200 truck drivers on highway during night are drunk. Is it valid to state that at least 90% of the truck drivers are drunk . [7M]