II B. Tech I Semester Supplementary Examinations, July - 2022
RANDAM VARIABLES AND STOCHASTIC PROCESSES
(Com to ECE, ECT)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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1 a) Write the properties of Gaussian density curve. Find the maximum value of Gaussian density function. [7M]

b) A random variable X has pdf $f_X(x) = k(1 + x^2)$, for $0 \le x \le 1$. Find the constant k and distribution function of random variable. [7M]

Or

2 a) Define and explain the following with an example: (i) Discrete sample space (ii) Conditional probability (iii) Continuous random variable. [7M]

b) The random variable X has the discrete variable in the set {-1, -0.5, 0.7, 1.5, 3} the corresponding probabilities are assumed to be {0.1, 0.2, 0.1, 0.4, 0.2}. Plot its distribution function and state is it a discrete or continuous distribution function. [7M]

3 a) Find the moment generating function of the random variable X whose moments are $m_r = (r + 1)!\, 2^r$ [7M]

b) State and prove the properties of variance of a random variable. [7M]

Or

4 a) What is meant by expectation? State and prove its properties. [7M]

b) A random variable X has pdf $f_X(x) = (1/b)e^{-(x-a)/b}$. Find its characteristic function. [7M]

5 a) State and prove the properties of joint density function. [7M]

b) Given the function $f_{XY}(x, y) = \begin{cases} b(x + y)^2, -2 < x < 2, -3 < y < 3; \\ 0, \quad elsewhere \end{cases}$ [7M]

Find the constant b such that this is a valid joint density function. Determine the marginal density functions.

Or

6 a) A joint sample space for two random variables X and Y has four elements (1,1), (2,2), (3,3) and (4,4). Probabilities of these elements are 0.1, 0.35, 0.05, and 0.5 respectively. (i)Determine through logic and sketch the distribution function $F_{XY}(x, y)$ (ii) Find the probability of the event {x≤2.5, y≤6} (iii) Find the probability of the event{x≤3} [7M]

b) Find the density function of W=X+Y, where the densities of X and Y are assumed to be: [7M]

$$f_X(x) = 0.5[u(x) - u(x - 2)];\ f_Y(y) = 0.25[u(y) - u(y - 4)]$$
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7 a) Define a random process. Write the classification of random process by the form of its sample functions and explain. [7M]

b) X (t) and Y (t) are real random processes that are jointly WSS. Prove the following (i)$R_{XY}(\tau) = \sqrt{R_{XX}(0)R_{YY}(0)}$ (ii)$R_{XY}(\tau) \leq \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$ [7M]

Or

8 a) Explain the following with respect to Random processes [7M]
(i) Strict sense stationarity
(ii) Mean Ergodic processes

b) Consider a random process $X(t) = A\, Cos(wt)$, where $'w'$ is a constant and A is a random variable Uniformly distributed over (0,1). Find the autocorrelation and auto - covariance of $X(t)$ [7M]

9 a) State and prove the relationship between Power Density Spectrum and Autocorrelation Function. [7M]

b) Define the following random processes [7M]
(i) Band pass process
(ii) Band limited process
(iii) Narrow band process

Or

10 a) If $X(t)$is a stationary process, find the power spectrum of $Y(t) = A_0 + B_0\, X(t)$ in term of the power spectrum of $X(t)$ if A_0 and $B_0$are real constants. [7M]

b) A random process $Y(t)$has the power spectral density [7M]

$$S_{YY}(\omega) = \frac{9}{\omega^2 + 64}$$

Find i) The average power of the process ii) The Auto correlation function.