

II B. Tech I Semester Supplementary Examinations, July - 2022
RANDOM VARIABLES AND STOCHASTIC PROCESSES
 (Com to ECE, ECT)

Time: 3 hours

Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

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- 1 a) Write the properties of Gaussian density curve. Find the maximum value of Gaussian density function. [7M]
 b) A random variable X has pdf $f_X(x) = k(1 + x^2)$, for $0 \leq x \leq 1$. Find the constant k and distribution function of random variable. [7M]

Or

- 2 a) Define and explain the following with an example: (i) Discrete sample space [7M]
 (ii) Conditional probability (iii) Continuous random variable.
 b) The random variable X has the discrete variable in the set $\{-1, -0.5, 0.7, 1.5, 3\}$ the corresponding probabilities are assumed to be $\{0.1, 0.2, 0.1, 0.4, 0.2\}$. Plot its distribution function and state is it a discrete or continuous distribution function. [7M]
- 3 a) Find the moment generating function of the random variable X whose moments are $m_r = (r + 1)! 2^r$ [7M]
 b) State and prove the properties of variance of a random variable. [7M]

Or

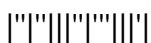
- 4 a) What is meant by expectation? State and prove its properties. [7M]
 b) A random variable X has pdf $f_X(x) = (1/b)e^{-(x-a)/b}$. Find its characteristic function. [7M]
- 5 a) State and prove the properties of joint density function. [7M]
 b) Given the function $f_{XY}(x, y) = \begin{cases} b(x + y)^2, & -2 < x < 2, -3 < y < 3; \\ 0, & elsewhere \end{cases}$ [7M]

Find the constant b such that this is a valid joint density function. Determine the marginal density functions.

Or

- 6 a) A joint sample space for two random variables X and Y has four elements (1,1), (2,2), (3,3) and (4,4). Probabilities of these elements are 0.1, 0.35, 0.05, and 0.5 respectively. (i) Determine through logic and sketch the distribution function $F_{XY}(x, y)$ (ii) Find the probability of the event $\{x \leq 2.5, y \leq 6\}$ (iii) Find the probability of the event $\{x \leq 3\}$ [7M]
 b) Find the density function of $W = X + Y$, where the densities of X and Y are assumed to be: [7M]

$$f_X(x) = 0.5[u(x) - u(x - 2)]; f_Y(y) = 0.25[u(y) - u(y - 4)]$$



- 7 a) Define a random process. Write the classification of random process by the form of its sample functions and explain. [7M]
- b) X (t) and Y (t) are real random processes that are jointly WSS. Prove the following [7M]
 (i) $R_{XY}(\tau) = \sqrt{R_{XX}(0)R_{YY}(0)}$ (ii) $R_{XY}(\tau) \leq \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$

Or

- 8 a) Explain the following with respect to Random processes [7M]
 (i) Strict sense stationarity
 (ii) Mean Ergodic processes
- b) Consider a random process $X(t) = A \cos(wt)$, where 'w' is a constant and A is a random variable Uniformly distributed over (0,1). Find the autocorrelation and auto - covariance of $X(t)$ [7M]
- 9 a) State and prove the relationship between Power Density Spectrum and Autocorrelation Function. [7M]
- b) Define the following random processes [7M]
 (i) Band pass process
 (ii) Band limited process
 (iii) Narrow band process

Or

- 10 a) If $X(t)$ is a stationary process, find the power spectrum of $Y(t) = A_0 + B_0 X(t)$ in term of the power spectrum of $X(t)$ if A_0 and B_0 are real constants. [7M]
- b) A random process $Y(t)$ has the power spectral density [7M]

$$S_{YY}(\omega) = \frac{9}{\omega^2 + 64}$$
 Find i) The average power of the process ii) The Auto correlation function.

