II B. Tech I Semester Regular/Supplementary Examinations, January-2023
RANDAM VARIABLES AND STOCHASTIC PROCESSES
(Com to ECE, ECT)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions, each Question from each unit
All Questions carry **Equal** Marks
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UNIT-I

1 a) Let $A_1, A_2, A_3, \ldots, A_n$ be a collection of mutually exclusive events whose union is S. If B be an event such that $P(B) \neq 0$, then find $P(B)$ in terms of elementary and conditional probabilities. [7M]

b) (i) Define Bernoulli random variable, X. Draw the typical CDF of X. [7M]
(ii) Consider the height of clouds is a Gaussian random variable, X, with $\mu_X = 1800$ and $\sigma_X = 450$. Plot $f_X(x)$. Also find the probability that the height of clouds is greater than 1850 meter.

OR

2 a) In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea? [7M]

b) Define the cumulative distribution function (CDF) of a random variable. List all the properties of CDF. [7M]

UNIT-II

3 a) Consider a random variable, X, with the PMF as tabulated below: [7M]

x	0	1	2	3
$p(x)$	1/8	1/8	1/4	1/2

Find
(i) mean value of X
(ii) variance of X

b) If $h_1(X)$ and $h_2(X)$ are two functions of a random variable X. Show that [7M]

$$E[c_1 h_1(X) + c_2 h_2(X)] = c_1 E[h_1(X)] + c_2 E[h_2(X)]$$

where c_1 and c_2 are real constants.

OR

4 a) If X is a random variable with mean, μ_X , then show that the standard deviation [7M]

$$\sigma_X = \sqrt{E[X^2] - (\mu_X)^2}$$

b) Find the characteristic function of a random variable with PDF [7M]

$$f_X(x) = \lambda e^{-\lambda x} u(x), \lambda > 0$$

UNIT-III

5 a) The calcium level in the blood, X, is between 8.5 and 10.5 milligrams per deciliter, and the cholesterol level, Y, is between 120 and 240 milligrams per deciliter. Assume that the joint density for X and Y is [7M]

$$f_{XY}(x, y) = c \qquad 8.5 \leq x \leq 10.5, 120 \leq y \leq 240$$

(i) Find the value of constant c.
(ii) Plot $f_{XY}(x, y)$.

b) Explain about Linear Transforms of Gaussian Random Variables?. [7M]

OR
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6 a) Explain how the univariate averages $E[X]$ and $E[Y]$ are computed via the joint density function when [7M]
   (i) X and Y are discrete random variables
   (ii) X and Y are continuous random variables

b) Show that if $X = Y$, then $Cov[X, Y] = Var[X] = Var[Y]$. [7M]

UNIT-IV

7 a) Distinguish between random variable and random process. Give suitable examples. [7M]

b) Two random processes $X(t)$ and $Y(t)$ are given by $X(t) = Acos(\omega_o t)$ and $Y(t) = Bsin(\omega_o t)$. Find the cross-correlation functions. [7M]

OR

8 a) Show that for jointly WSS real random processes $X(t)$ and $Y(t)$, [7M]
$$|R_{XX}(\tau)| \leq [R_{XX}(0)R_{YY}(0)]^{1/2}$$

b) Given a random process $X(t) = kt$, where k is a random variable uniformly distributed in the range (-1, 1). Is the process WSS? [7M]

UNIT-V

9 a) Consider the LTI system in Fig.1, with WSS random process $X(t)$. Find the mean-value of output random process, (t). [7M]

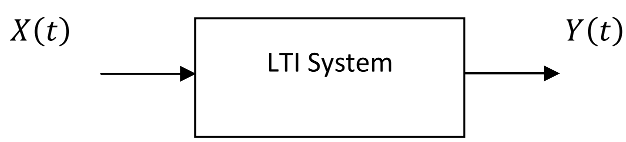

Fig.1

b) State all the properties of cross-spectral densities. [7M]

OR

10 a) Consider the LTI system shown in Fig.2, with $S_{XX}(\omega) = \frac{1}{1+\omega^2}$. Find the average power of output random process, $Y(t)$. [7M]

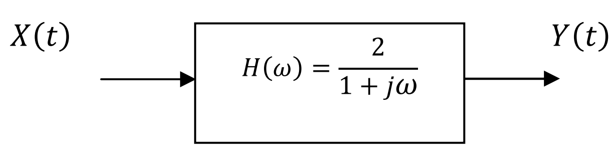

Fig.2

b) Show that $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-j\omega\tau}d\tau$. [7M]

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UNIT-I

1 a) (i) Define uniform random variable. [7M]
(ii) Give the mathematical expressions for CDF and PDF of uniform random variable.
(iii) Plot the CDF and PDF of uniform random variable.
b) State and prove total probability theorem. [7M]

OR

2 a) What do you understand by mathematical modeling of a random experiment? Explain with an example. [7M]
b) Suppose the waiting time of data packets in a computer network is an exponential random variable with PDF $f_X(x) = 0.5\exp(-0.5x)\,u(x)$. [7M]
(i) Plot $f_X(x)$
(ii) Find the $P(0.1 < X \leq 0.5)$.

UNIT-II

3 a) Find the variance of uniform random variable. [7M]
b) Consider a discrete random variable with PDF [7M]
$$f_X(x) = p_1\delta(x-1) + p_2\delta(x-2), 0 \leq p_1 \leq 1, 0 \leq p_2 \leq 1$$
Find $f_Y(y)$, if $Y = X + 1$.

OR

4 a) Show that $E[X^n]$ can be computed from the characteristic function of a random variable. [7M]
b) State and prove the properties of variance of a random variable. [7M]

UNIT-III

5 a) The joint density function of two random variables is given by [7M]
$$f_{XY}(x,y) = \frac{1}{2(e-1)}\left[\frac{1}{x} + \frac{1}{y}\right];\ 1 \leq x \leq e, 1 \leq y \leq e$$
Find $\int_1^e \int_1^e f_{XY}(x,y)dydx$. Comment on the result.
b) Consider the linear transformation T defined by [7M]
$$T: \quad u = 2x + y, v = x + 3y$$
(i) Is this transformation invertible? If so, find the defining equations for T^{-1}.
(ii) Find the Jacobian for T^{-1}.

OR
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6 a) Assume that $Y = a + bX, b \neq 0$. Show that $Cov[X, Y] = b\, Var[X]$. [7M]

b) List the properties of jointly Gaussian random variables. [7M]

UNIT-IV

7 a) Define the following: [7M]
(i) First order stationarity
(ii) Second order stationarity
(iii) N^{th} order stationarity
(iv) Wide-sense stationarity

b) Give the classification of random processes based on statistical properties. [7M]

OR

8 a) Give an example of a random process that satisfies the following: [7M]
(i) Mean ergodicity
(ii) Auto-correlation ergodicity

b) State all the properties of autocorrelation function. [7M]

UNIT-V

9 a) Discuss about power density spectrum properties. [7M]

b) Explain the relationship between Cross power density spectrum and Cross correlation function. [7M]

OR

10 a) Draw the power spectrum of bandpass random process. Derive the auto-correlation function from it. [7M]

b) Derive the relation between $R_{YY}(\tau)$ and $R_{XX}(\tau)$ corresponding to the following LTI system in Fig.1. [7M]

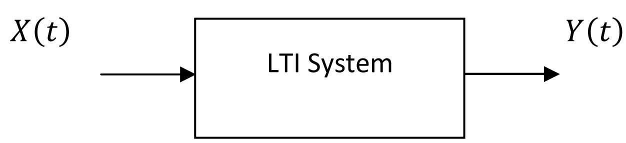

Fig.1

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UNIT-I

1 a) Define conditional distribution function, and list all its properties. [7M]

b) Define Gaussian random variable, X. Draw the PDF of X for two different values of σ, i.e., σ_1 and σ_2, assuming $\sigma_1 < \sigma_2$. What is your observation? [7M]

OR

2 a) Explain the following: (i) Point conditioning (ii) Interval conditioning [7M]

b) Suppose there is an error probability of 0.05 per word in typing using an electronic type-writer machine. What is the probability that there will be more than one error in a page of 120 words? [7M]

UNIT-II

3 a) If $Y = aX + b$, where a and b are real constants, find the variance of Y. [7M]

b) The characteristic function of a random variable is given by [7M]

$$\Phi_X(\omega) = \frac{1}{1 - j\omega}$$

Find
(i) the mean value of X
(ii) variance of X

OR

4 a) Define the following: [7M]
(i) Skew
(ii) Coefficient of skewness
(iii) n^{th} central moment

b) A random variable X is uniformly distributed over the interval (0, 1). Find the PDF of a new random variable, $Y = X^2$. [7M]

UNIT-III

5 a) Let U and V have the joint PDF: [7M]

$$f_{UV}(u,v) = \begin{cases} u + v\,, 0 \le u, v \le 1 \\ 0, \quad elsewhere \end{cases}$$

Given that $X = U^2$ and $Y = U(1 + V)$. Find the joint PDF of X and Y.

b) Define the following: [7M]
(i) Joint moments about the origin
(ii) Joint central moments
(iii) Correlation coefficient

OR
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6 a) Define joint characteristic function. Explain how the joint moments are obtained from joint characteristic function. [7M]

b) The joint PDF of X and Y is given by $f_{XY}(x,y) = \begin{cases} 4x^2, 0 < x, y < 1 \\ 0, elsewhere \end{cases}$ [7M]

Find (i) $E[XY]$ and (ii) $f_Y(y)$.

UNIT-IV

7 a) Consider a random process $X(t) = Acos(\omega_o t + \Theta)$. A and ω_o are real constants, and Θ is uniformly distributed over (-π, π). Verify that $X(t)$ is WSS. [7M]

b) If $X(t)$ is WSS random process, verify that $|R_{XX}(\tau)| \leq E[X^2(t)]$. [7M]

OR

8 a) Prove that $|R_{XX}(\tau)| \leq \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$. [7M]

b) Derive the relation between auto-correlation function and auto-covariance function of a random process. [7M]

UNIT-V

9 a) A WSS white noise process of PSD $\frac{N_o}{2}$ is input to a first-order RC lowpass filter. Find the output variance. [7M]

b) List the properties of [7M]

(i) Power spectral density of a random process, $X(t)$

(ii) Cross power spectral densities of random processes $X(t)$ and $Y(t)$

OR

10 Derive the relation between PSD of output and PSD of input of an LTI system. [14M]

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UNIT-I

1 a) (i) Define probability density function of a random variable. [8M]
(ii) Show that the area under probability density function is unity.
b) Define the following and give one example for each: [6M]
(i) Statistically independent events
(ii) Mutually exclusive events
(iii) Discrete sample space

OR

2 a) (i) Give the concept of random variable with an example. [8M]
(ii) What are the conditions to be satisfied for a function to be random variable?
b) Define the following and give one example for each: [6M]
(i) Continuous sample space
(ii) Exhaustive events
(iii) Equally likely events

UNIT-II

3 a) Show that the variance of a uniform random variable, $U(a,b)$, is $\frac{(b-a)^2}{12}$. [7M]
b) Let X is a random variable. Find the density function of [7M]
$$Y = exp(X)$$
Carefully plot $f_Y(y)$.

OR

4 a) Consider the random variable X with probability density function [7M]
$$f_X(x) = \begin{cases} \left(\frac{1}{6}\right)x, & 2 \le x \le 4 \\ 0, & otherwise \end{cases}$$
Find (i) $E[X]$, (ii) $E[X^2]$ and (iii) σ
b) Let X be a continuous random variable with density $f_X(x)$, and a new random variable is formed by the transformation [7M]
$$Y = X^2$$
Show that for $y \ge 0$, $F_Y(y) = F_X(\sqrt{y}) - F_X(-\sqrt{y})$.

UNIT-III

5 a) The joint density of two random variables is given by [7M]
$$f_{XY}(x,y) = \frac{1.72}{x}, 27 \le x, y \le 33$$
Find the marginal densities $f_X(x)$ and $f_Y(y)$.
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b) Explain how $E[X]$, $E[Y]$, $E[X^2]$ and $E[Y^2]$ are computed using joint probability density function of two random variables X and Y. [7M]

OR

6 a) The joint density of two random variables is given by [7M]

$$f_{XY}(x,y) = \begin{cases} \frac{1}{6}; & 0 < x < 2, 0 < y < 3 \\ 0, & elsewhere \end{cases}$$

Find the joint density of U and V, when $U = X - Y$ and $V = X + Y$.

b) Define the bivariate Gaussian random variable. List all the properties of jointly Gaussian random variables. [7M]

UNIT-IV

7 a) List all the properties of autocorrelation function. [7M]

b) Give the classification of random variables based on the type of random variable and time. [7M]

OR

8 a) State and prove all the properties of cross-correlation functions. [7M]

b) What is ergodicity? Explain the concept of mean-ergodicity and autocorrelation-ergodicity with an example. [7M]

UNIT-V

9 a) Show that the autocorrelation function and power spectral density form Fourier transform pair. [7M]

b) Find the power spectral density and average power of $X(t)$ with [7M]

$$R_{XX}(\tau) = \exp(-|\tau|)$$

OR

10 a) Show that the output random process of an LTI system is also WSS process, when the input random process is a WSS process. [7M]

b) Define the following: [7M]

(i) Bandpass process
(ii) Narrowband process
(iii) Bandlimited process