II B. Tech I Semester Regular/Supplementary Examinations, December-2023
RANDOM VARIABLES AND STOCHASTIC PROCESSES
(Com to ECE, ECT)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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UNIT-I

1 a) State and prove the properties of cumulative distribution function (CDF) of X. [7M]

b) The random variable X has the discrete variable in the set {-1, -0.5, 0.7, 1.5, 3} the corresponding probabilities are assumed to be {0.1, 0.2, 0.1, 0.4, 0.2}. Plot its distribution function and state is it a discrete or continuous distribution function. [7M]

OR

2 a) (i) Define probability density function of a random variable. [7M]
(ii) Show that the area under probability density function is unity

b) Suppose there is an error probability of 0.05 per word in typing using an electronic type-writer machine. What is the probability that there will be more than one error in a page of 120 words? [7M]

UNIT-II

3 a) Consider the random variable X with probability density function [7M]

$$f_X(x) = \begin{cases} \left(\frac{1}{6}\right)x, & 2 \le x \le 4 \\ 0, & otherwise \end{cases}$$

Find (i) E[X], (ii)$E[X^2]$ and (iii)σ

b) If $h_1(X)$ and $h_2(X)$ are two functions of a random variable X . Show that $E[c_1h_1(X)+ c_2h_2(X)] = c_1E[h_1(X)] + c_2E[h_2(X)]$
Where $c_1$and c_2 are real constants. [7M]

OR

4 a) Consider a random variable, X, with the PMF as tabulated below [7M]

x	0	1	2	3
$p(x)$	1/8	1/8	1/4	1/2

Find
(i) mean value of X
(ii) variance of X

b) Let X be a continuous random variable with density $f_X(x)$, and a new random variable is formed by the transformation $Y=X^2$ [7M]
Show that for y≥0, $F_Y(y) = F_X(\sqrt{y}) - F_X(-\sqrt{y})$

UNIT-III

5 a) State and prove the properties of joint density function [7M]
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	b)	A joint sample space for two random variables X and Y has four elements (1,1), (2,2), (3,3) and (4,4). Probabilities of these elements are 0.1, 0.35, 0.05, and 0.5 respectively. (i)Determine through logic and sketch the distribution function $F_{XY}(x,y)$ (ii) Find the probability of the event $\{x \le 2.5, y \le 6\}$ (iii) Find the probability of the event $\{x \le 3\}$	[7M]
		OR	
6	a)	Define the bivariate Gaussian random variable. List all the properties of jointly Gaussian random variables.	[7M]
	b)	The joint density of two random variables is given by $f_{XY}(x,y) = \begin{cases} \frac{1}{6}; & 0 < x < 2, 0 < y < 3 \\ 0, & elsewhere \end{cases}$ Find the joint density of U and V, when $U = X - Y$ and $V = X + Y$	[7M]
		UNIT-IV	
7	a)	List all the properties of autocorrelation function.	[7M]
	b)	What is ergodicity? Explain the concept of mean-ergodicity and autocorrelation-ergodicity with an example.	[7M]
		OR	
8	a)	Explain about Poisson random processes.	[7M]
	b)	Derive the relation between correlation and covariance of two random variables X and Y.	[7M]
		UNIT-V	
9	a)	Find the mean and mean- square values of output y(t) of an LTI system with input x(t). Assume that x(t) is a WSS process.	[7M]
	b)	Find the power spectral density and average power of X(t) with $R_{XX}(\tau) = \exp(-\lvert\tau\rvert)$	[7M]
		OR	
10	a)	Define the following systems with applications. (i) Band –Limited process (ii)Band – Limited Band pass process	[7M]
	b)	Show that the autocorrelation function and power spectral density form Fourier transform pair.	[7M]

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UNIT-I

1 a) Define and explain the following with an example: (i) Discrete sample space (ii) Conditional probability (iii) Continuous random variable. [7M]

b) Given that a random variable X has the following possible values, state if X is discrete, continuous or mixed [7M]
i. {-20<x<-5}
ii. {10,12<x<=14,15,17)
iii. {-10 for s>2 and 5 for s<=2, where 1<s<=6}
iv. {4,3.1,1, -2}

OR

2 a) Define the following and give one example for each: [7M]
(i) Statistically independent events
(ii) Mutually exclusive events
(iii) Discrete sample space

b) Two boxes are selected randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball? [7M]

UNIT-II

3 a) Show that the variance of a uniform random variable, U(a, b), is $\frac{(b-a)^2}{12}$ [7M]

b) Let X is a random variable. Find the density function of Y=exp(X) Carefully plot $f_Y(y)$ [7M]

OR

4 a) Show that $E[X^n]$ can be computed from the characteristic function of a random variable. [7M]

b) If $Y = aX+b$, where a and b are real constants, find the variance of Y. [7M]

UNIT-III

5 a) Explain how E[X], E[Y], $E[X^2]$ and $E[Y^2]$ are computed using joint probability density function of two random variables X and Y [7M]

b) Show that if $X=Y$, then $Cov[X,Y]=Var[X]=Var[Y]$. [7M]

OR
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6 a) Explain about Linear Transforms of Gaussian Random Variables. [7M]

b) Explain how the univariate averages E[X]and E[Y]are computed via the joint density function when [7M]
(i) X and Y are discrete random variables
(ii) X and Yare continuous random variables

UNIT-IV

7 a) List all the properties of autocorrelation function. [7M]

b) Two random processes $X(t)$ and $Y(t)$are given by $X(t) = Acos(\omega_o t)$ and $Y(t) = Bsin(\omega_o t)$. Find the cross-correlation functions. [7M]

OR

8 a) Give the classification of random variables based on the type of random variable and time. [7M]

b) Given a random process X(t) = kt, where A is a random variable uniformly distributed in the range (-1, 1). Is the process WSS? [7M]

UNIT-V

9 a) Derive the relationship between cross-power spectral density and cross correlation function. [7M]

b) If $X(t)$ is a stationary process, find the power spectrum of $Y(t)=A_0+B_0 X(t)$ interms of the power spectrum of $X(t)$ if A_0 and B_0 are real constants. [7M]

OR

10 a) Show that the output random process of an LTI system is also WSS process, when the input random process is a WSS process. [7M]

b) State all the properties of cross-spectral densities. [7M]

Code No: R2021044 **R20** SET - 3

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UNIT-I

1 a) Define conditional probability distribution function and write the properties. [7M]

b) Let A1, A2, A3,….., An be a collection of mutually exclusive events whose union is S. If B is an event such that P(B) ≠ 0, then find P(B) in terms of elementary and conditional probabilities. [7M]

OR

2 a) State and prove total probability theorem. [7M]

b) In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea? [7M]

UNIT-II

3 a) State and prove the properties of variance of a random variable. [7M]

b) A random variable X is uniformly distributed over the interval (0, 1). Find the PDF of a new random variable, $Y=X^2$ [7M]

OR

4 a) Define the following: [7M]
(i) Skew
(ii) Coefficient of skewness
(iii) n^{th} central moment

b) If $Y = aX+b$, where a and b are real constants, find the variance of Y. [7M]

UNIT-III

5 a) List the properties of jointly Gaussian random variables. [7M]

b) Show that if$X=Y$, then $Cov[X,Y]=Var[X]=Var[Y]$. [7M]

OR

6 a) Define joint characteristic function. Explain how the joint moments are obtained from joint characteristic function. [7M]

b) The joint density function of two random variables is given by [7M]
$f_{XY}(x,y) = \frac{1}{2(e-1)}\left[\frac{1}{x}+\frac{1}{y}\right];\ 1 \le x \le e, 1 \le y \le e$ Find $\int_1^e \int_1^e f_{XY}(x,y)dydx$
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UNIT-IV

7 a) Define the following: [7M]
(i) First order stationarity
(ii) Second order stationarity
(iii) Nth order stationarity
(iv) Wide-sense stationarity

b) Prove that $|R_{XX}(\tau)| \leq \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$ [7M]

OR

8 a) Give the classification of random processes based on statistical properties. [7M]

b) What is ergodicity? Explain the concept of mean-ergodicity and autocorrelation-ergodicity with an example. [7M]

UNIT-V

9 a) Find the mean and mean- squave values of output y(t) of an LTI system with input x(t). Assume that x(t) is a WSS process. [7M]

b) If X(t) is a stationary process, find the power spectrum of) $Y(t) = A0 + B0\,X(t)$ in term of the power spectrum of X(t) if $A0$ and $B0$ are real constants. [7M]

OR

10 a) Derive the relationship between cross-power spectral density and cross correlation function. [7M]

b) Find the power spectral density and average power of X(t) with $R_{XX}(\tau) = \exp(-|\tau|)$ [7M]

Code No: R2021044 **R20** SET - 4

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UNIT-I

1 a) Explain the following: (i) Point conditioning (ii) Interval conditioning [7M]

b) Two boxes are selected randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball? [7M]

OR

2 a) (i) Define uniform random variable. [7M]
(ii) Give the mathematical expressions for CDF and PDF of uniform random variable.
(iii) Plot the CDF and PDF of uniform random variable.

b) Suppose the waiting time of data packets in a computer network is an exponential random variable with PDF $f_X(x)=0.5\exp(-0.5x)u(x)$. [7M]
(i) Plot $f_X(x)$
(ii) Find the $P(0.1 < X \leq 0.5)$.

UNIT-II

3 a) Find mean and variance of Gaussian random variable. [7M]

b) A Gaussian random variable with variance 10 and mean 5 is transformed to $y=e^x$. Find the pdf of y. [7M]

OR

4 a) State and prove the Chebychev's inequality theorem. [7M]

b) Show that any characteristic function $\Phi_X(\omega)$ satisfies $\Phi_X(\omega)\leq\Phi_X(0)=1$ [7M]

UNIT-III

5 a) Assume that $Y= a+bX,\ b\neq 0$. Show that $Cov[X,Y]= bVar[X]$. [7M]

b) The joint density function of two random variables is given by [7M]
$f_{XY}(x,y) = \frac{1}{2(e-1)}\left[\frac{1}{x}+\frac{1}{y}\right];\ 1\leq x\leq e, 1\leq y\leq e$ Find $\int_1^e\int_1^e f_{XY}(x,y)\,dy\,dx$

OR

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6 a) Define the following: [7M]
(i) Joint moments about the origin
(ii) Joint central moments
(iii) Correlation coefficient

b) The joint density of two random variables is given by [7M]

$$f_{XY}(x,y) = \begin{cases} \frac{1}{6}; & 0 < x < 2, 0 < y < 3 \\ 0, & elsewhere \end{cases}$$

Find the joint density of U and V, when $U = X - Y$ and $V = X + Y$

UNIT-IV

7 a) Give an example of a random process that satisfies the following: [7M]
(i) Mean ergodicity
(ii) Auto-correlation ergodicity

b) Prove that $|R_{XX}(\tau)| \leq \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$ [7M]

OR

8 a) State all the properties of autocorrelation function. [7M]

b) Derive the relation between correlation and covariance of two random variables X and Y. [7M]

UNIT-V

9 a) Show that $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$ [7M]

b) Derive the relation between PSD of output and PSD of input of an LTI system. [7M]

OR

10 a) Show that the autocorrelation function and power spectral density form Fourier transform pair. [7M]

b) Define the following random processes [7M]
(i) Band pass process
(ii) Band limited process
(iii) Narrow band process