II B. Tech II Semester Regular/Supplementary Examinations, July - 2023
COMPLEX VARIABLES AND STATISTICAL METHODS
(Common to CE, ME, AME & MM)

Time: 3 hours   Max. Marks: 70

Answer any **FIVE** Questions, each Question from each unit
All Questions carry **Equal** Marks
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UNIT-I

1. a) If f (z) is regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ [7M]

   b) Find the value of the integral $\int_0^{1+i}(x - y + ix^2)dz$ along the real axis from z=0 to z=1 and then along a line parallel to the imaginary axis from z=1 to z=1+i   [7M]

Or

2. a) Find the regular function whose imaginary part is $Log(x^2 + y^2) + x - 2y$.   [7M]

   b) Evaluate the integral $\int_C Tanz\, dz$ where C is $|z| = 2$ [7M]

UNIT-II

3. a) Represent the function f (z) $=\frac{z+3}{z(z^2-z-2)}$ in Laurent series i) within$|z| = 1$ ii)Exterior to $|z| = 3$ [7M]

   b) Evaluate $\int_C \frac{z^3}{(z-1)^2(z-3)}\, dz$ where C is $|z| = 2$ by residue theorem.   [7M]

Or

4. a) The only singularities of a single valued function f(z) are poles of order 2 and 1 at z =1 & z = 2 with residues of these poles as 1 and 3 respectively. If f (0) $=\frac{3}{2}$, f (-1) = 1, determine the function.   [7M]

   b) Using the method of contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2}\, dx$ [7M]

UNIT-III

5. a) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find (i) How many students score between 12 and 15? (ii)How many score above 18?(iii) How many score below 18   [7M]

   b) Is $f(x) = \frac{1}{2}x^2 e^{-x}$ when x ≥ 0 can be regarded as a probability function for a continuous random variable? If, so find Mean and Variance of the random variable.   [7M]

Or

6. a) The density function of a random variable X is given by f(x) = k x (2-x), $0 \leq x \leq 2$. Find (i) k (ii) mean (iii) variance.   [7M]

   b) In a test on electrical bulbs, it was found that the life of a particular make is normally distributed with an average life time of 2040 hours and Standard deviation of 40 hours. Estimate the number of bulbs likely to burn for more than 2140 hours in a sample of 2000 bulbs.   [7M]

UNIT-IV

7 a) Find 95% confidence limits for the mean of a normality distributed population from which the following sample was taken and taken 15, 17,10,18,16,9,7,11,13,14.   [7M]

   b) An oceanographer wants to check whether the depth of the ocean in a certain region is 57.4fathoms, as had previously been recorded. What can he conclude at 0.05 level of significance, if reading taken at 40 random locations in the given region yielded a mean of 59.1 fathoms with a standard deviation of 5.2 fathoms.   [7M]

Or

8 a) A population consists of six numbers 4,8,12,16,20,24. Consider all samples of size two which can be drawn without replacement from this population.   [7M]
      i.   The mean of the sampling distribution of means.
      ii.   The standard deviation of the sampling distribution of means.

   b) Define estimation and write different types of estimations.   [7M]

UNIT-V

9 a) Write about (i) Null Hypothesis and Alternate Hypothesis (ii) Type I and Type II errors (iii) Critical region.   [7M]

   b) At a certain large university a sociologist speculates that male students spend considerably more   money on junk food than do female students. To test her hypothesis, the sociologist randomly selects from the registrars' records the names of 200 students. Of these 125 are men and 75are women. The average amount spent on junk food per week by the men is Rs.400/-   and standard deviation is Rs100/-. For the women, the sample mean is Rs.450/- and the S.D is Rs.150/-.Test the difference between the mean at 0.05 level of significance.   [7M]

Or

10 a) Two sets of 100 students each were taught to read by two different methods. After the instructions were over, a reading test given to them revealed that $\overline{x_1} = 73.4$, $\overline{x_2} = 70.3$, $s_1 = 8$ $s_2 = 10$.Test the hypothesis that $\mu_1 = \mu_2$.   [7M]

   b) It is known that the mean diameters of rivets produced by two firms A and B are 53 and 48 respectively, and the standard deviations for 42 rivets produced by firm A, the standard deviation is 2.9mm, while 45 rivets manufactured by firm B, the standard deviation is 3.8mm. Compute the statistic you would use to test whether the products of firm A have the same as those of firm B and test its significance.   [7M]

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UNIT-I

1. a) If u+i v is an analytic function, prove that u (x, y) =k; v(x, y) =c denote orthogonal families of curves. [7M]

   b) Evaluate $\int_C [(2x - 3y + 4) + i(2y + 3x)]dz$ where C is the arc along $y = 4x + 1$ from the point (0,5) to (4,17). [7M]

Or

2. a) Using Cauchy's integral formula find the value of $\int_C \frac{f(z)}{(z-2)} dz$, where C is the region $\frac{3}{2} < |z| < \frac{5}{2}$ and f(z)=exp(z). [7M]

   b) If $P(x,y) = e^x Cosy$,then find the function Q(x, y) such that P+iQ is an analytic function [7M]

UNIT-II

3. a) Find the Laurent series expansion of the function $\frac{z^2-1}{(z+3)(z+2)}$ in the range $2 < |z| < 3$ [7M]

   b) Define (i) singular point. (ii)Explain types of singular points and give one example for each. [7M]

Or

4. a) Expand $f(z) = \frac{z-1}{z^2}$ in Taylor's series in powers of $(z - 1)$ and determine the region of convergence. [7M]

   b) Show that $\int_0^\infty \frac{sin\,xdx}{x} = \frac{\pi}{2}$, using complex integration. [7M]

UNIT-III

5. a) In a certain college 25% of boys and 10% of girls are studying Mathematics. The girls constitute 60% of the student body.
   (i) What is the probability that mathematics is being studied?
   (ii) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl? [7M]

   b) Let X be a random variable defined by the density function [7M]
   f(X=x) =$\begin{cases} \frac{\pi}{6}\sqrt{k}\,cos\frac{\pi x}{6}, -4 \le x \le 4 \\ 0, other\,wise \end{cases}$, Find k and E[3X].

Or

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6   a)   i) Define Random variable and explain the two types of random variables with examples for each (ii) Define mean and variance of the probability distributions of the discrete and continuous random variables   [7M]

   b)   If X is a normal random variable with mean 26 and S.D. 4. Find the probability that i) $23 \le X \le 30$ ii) $18 \le X \le 24$.   [7M]

UNIT-IV

7   a)   A population consists of six numbers 2, 3, 6, 8, 11. Consider all samples of size two which can be drawn with replacement from this population.   [7M]
      i.   The mean of the sampling distribution of means.
      ii.   The standard deviation of the sampling distribution of means.

   b)   A random sample of 400 items is found to have mean of 82 and S.D of 18.find 95% confidence Limits for the mean of the population from which the sample is drawn   [7M]

Or

8   a)   The 9 items of a sample have the values 45, 47, 50,52,48,47,49,53,51. Find the 95% confidence interval.   [7M]

   b)   A random sample of size 81 was taken whose variance is 20.25 and means 32 construct 98% confidence limits.   [7M]

UNIT-V

9   a)   In a random sample of 1000 persons from town A, 400 are found to be consumers of wheat. In a random of 800 from town B, 400 are found to be consumers of wheat, do these data reveal a significant difference between town A and town B, so far as the proportion of wheat consumers is concerned?   [7M]

   b)   Explain the test procedure for testing of hypothesis.   [7M]

Or

10 a)   A company is interested in knowing if there is a difference in the average salary received by foremen in two divisions. Accordingly samples of 12 foremen in the first division and 10 in the second division are selected at random. The data is given as follows.   [7M]

	First division	Second division
Sample size	12	10
Average monthly salary(in Rs)	1050	980
Standard deviation of salaries (in Rs)	68	74

Test the significance difference between means at 5% Level of significance

   b)   A die was thrown 9000 times and a throw of 5or 6 was obtained 3240 times. On the assumption of random throwing do the data indicate an unbiased die?   [7M]

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SET - 3

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UNIT-I

1. a) If f(z) is an analytic function, find $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) log|f'(z)|$ [7M]

   b) Using Cauchy's integral formula, evaluate $\int_C \frac{e^{-z}}{(z+2)^5} dz$ where C is the circle $|z| = 3$ [7M]

Or

2. a) Find the Analytic function f(z)=u+iv in terms of z, where [7M]
$$u - v = e^x(cos\, y - sin\, y)$$

   b) Evaluate $\int_C [(3x^2 - 4xy - 3y^2) + i2(x^2 + 3xy - y^2)]dz$ where C is the arc along $y = x^2$ from the point 0 to (1+i) [7M]

UNIT-II

3. a) Represent the function f (z) = $\frac{1}{z(z+2)^3(z+1)^2}$ in Laurent series within [7M]
$$\frac{5}{4} \le |z| \le \frac{7}{4}$$

   b) Using complex variable techniques evaluate $\int_0^{2\pi} \frac{sin^2\,\theta\, d\theta}{5 - 4\, cos\, \theta}$. [7M]

Or

4. a) Find the Laurent series expansion of $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region [7M]
$3 < |z + 2| < 5$.

   b) Evaluate $\oint_C \frac{z-3}{z^2+2z+5} dz$ where C is the contour $|z + 1 - i| = 2$, using residue theorem [7M]
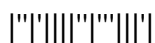
UNIT-III

5. a) In a class, 2% of the boys and 3% of girls are having blue eyes. There are 70% boys in the class. A student is selected and having blue eyes. What is the probability that the student is girl? [7M]

   b) State Bayes theorem. Three urns A, B, C contains white, red and green balls as given below. Two balls are drawn from an urn chosen at random. These are found to be one white and one Green. Find the probability that the balls drawn are from urn C. [7M]

	Urn A	Urn B	Urn C
White	1	2	4
Red	2	1	5
Green	3	1	3

Or

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6 a) Write Probability distribution of the random variable of having number of red color cards when 3 cards are drawn from a pack of 52 cards. Find the mean of the distribution. [7M]

   b) In a Poisson distribution $P(X=x)$ for $x=0$ is 10%. Find the mean and $P(X<3)$, (Log 10 = 2.3026.) [7M]

UNIT-IV

7 a) A random sample of size 81 whose variance is 26.25 and mean is 32. Construct a 98% confidence interval. [7M]

   b) Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from a distribution with mean and standard deviation μ, σ respectively. Show that $S^2 = \sum \frac{(X_i - \bar{X})^2}{n-1}$ is unbiased estimator of variance. [7M]

Or

8 a) A population consists of six numbers 1, 2, 3, 4, 6, 8. Consider all samples of size two which can be drawn without replacement from this population. [7M]

         i. The mean of the sampling distribution of means.

         ii. The standard deviation of the sampling distribution of means.

   b) The mean height of students in a college is 155 cm and S.D. is 15cm. What is the probability that the mean height of 36 students is less than 157 cm. [7M]

UNIT-V

9 a) Explain one-tailed test and two tailed test with an example each. [7M]

   b) An ambulance service claims that it takes on the average less than 10 minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and the variance of16 minutes. Test the significance at 0.05 level. [7M]
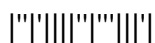
Or

10 a) Two animals C and B were tested according to the time to run a particular track with the following results: [7M]

Animal C	28	29	33	32	30	34
Animal B	27	30	32	33	30	28

Test whether the two animals have the same running capacity.

   b) The average marks in mathematics of a sample of 100 students were 51 with a standard deviation of 6 marks. Could this have been a random sample from a population with average marks of 50. [7M]

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UNIT-I

1 a) Show that the function defined by f(z)=$\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ at z≠0 and f (0) =0 is continuous and satisfy C-R equations at origin but $f^/(0)$ does not exist. [7M]

   b) Evaluate $\int_C \frac{z}{z^2+1}dz$ where c is $\left|z+\frac{1}{z}\right|=2$. [7M]

Or

2 a) Find the analytic function f (z) = u+iv in terms of z whose real part u = 3x-2xy. [7M]

   b) Evaluate $\int_{-1+i}^{1+i}(y+ix^2)dz$ along the parabola $x^2=y$. [7M]

UNIT-II

3 a) Expand $f(z)=\frac{e^z}{z(z+1)}$ as a Taylor's series about z=2. [7M]

   b) Evaluate $\int_0^{2\pi}\frac{d\theta}{a+\cos\theta}$ using residue theorem. [7M]

Or

4 a) Find the residue of $f(z)=\frac{(z^2+1)}{(z^2+2z+3)^2}$ at the pole above the real axis. [7M]

   b) Using complex variable techniques evaluate $\int_0^{2\pi}\frac{\sin^2\theta d\theta}{5-4\cos\theta}$. [7M]

UNIT-III

5 a) State and prove Baye's theorem. [7M]

   b) For the discrete probability distribution [7M]

X	0	1	2	3	4	5	6
F	0	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find (i) k (ii) Mean (iii) Variance

Or

6 a) In a factory machines A, B, C manufacture 25%, 35% and 40% of the total their output and 6%, 3%, and 2% are defective. A bolt is drawn at random. If the selected bolt found to be defective, what is the probability that it is manufactured by machine C. [7M]

   b) If X is a normal variate with mean 32 and S.D. 5.5. Find the probability that i) $24\le X\le 30$ ii) $26\le X\le 40$. [7M]
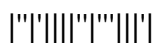
UNIT-IV

7 a) What is the effect on standard error, if a sample is taken from an infinite population of sample size is (i) Increased from 50 to 200 (ii) Decreased from 640 to 40 [7M]

b) The guaranteed average life of a certain type of electric bulbs is 1500 hrs with a S.D. of 120 hrs. It is decided to sample the output so as ensure that 95% of bulbs do not fall short on the guaranteed average by more than 2%. What will be the minimum sample size? [7M]

Or

8 a) Explain Point estimation and interval estimation with examples. [7M]

b) A population consists of six numbers 3, 6, 9, 15, 27. Consider all samples of size two which can be drawn with replacement from this population. [7M]
　　　　i. The mean of the sampling distribution of means.
　　　　ii. The standard deviation of the sampling distribution of means.

UNIT-V

9 a) The nicotine in milligrams of two samples of tobacco were found to be as follows. Test the significant difference between the means are equal or not at 0.05 level [7M]

Sample A	24	27	26	23	25	-
Sample B	29	30	30	31	24	30

b) It is claimed that a random sample of 100 tyres with a mean life of 15269 km is drawn from a population of tyres which has a mean life of 15200 km and a standard deviation of 1248 km. Test the validity of the claim at 5% LOS. [7M]

Or

10 a) A company claims that its light bulbs are superior to those of the competitor. If a study showed that a sample of 40 of its bulbs has a mean life time of 647 hrs of continuous use with a Standard deviation of 27 hours. While a sample of 40 bulbs made by its main competitor had a mean life time of 638hours of continuous use with a S.D of 31 hours. Test the significance between the differences of the two means at5% LOS. [7M]

b) A random sample of 8 girls have the following I.Q.'s : 70, 120, 110, 100, 107, 95, 98, 88. Do these data support the assumption of a population mean I.Q. of 102. [7M]

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