

II B. Tech II Semester Regular Examinations, June/July - 2022
COMPLEX VARIABLES AND STATISTICAL METHODS
 (Common to CE, ME, AME & MM)

Time: 3 hours

Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

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**UNIT-I**

- 1 a) Show that for  $f(z) = \begin{cases} \frac{(x^3 - y^3) + i(x^3 + y^3)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$  the Cauchy-Riemann [10M]  
 equations are satisfied at the origin but the derivative of  $f(z)$  at origin does not exist.
- b) Evaluate  $\int_0^{3+i} z^2 dz$  along the paths the real axis to 3 and then vertically to  $3+i$ . [4M]

**Or**

- 2 a) Find the analytic function  $f(z) = u + iv$  where  $v = e^x \sin y$ . [7M]  
 b) Use Cauchy's integral formula to evaluate  $\oint_C \frac{1}{z^2 + 9} dz$  where C is the circle (i) [7M]  
 $|z - 3i| = 4$  (ii)  $|z + 3i| = 2$ .

**UNIT-II**

- 3 a) Find the Laurent series of  $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$ , for  $1 < |z| < 4$ . [7M]  
 b) Use residue theorem to evaluate  $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin \theta}$ . [7M]

**Or**

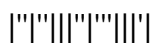
- 4 a) Find Taylor's expansion of  $f(z) = \log(1+z)$  about the point  $z = 0$ . [6M]  
 b) Use residue theorem to evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ . [8M]

**UNIT-III**

- 5 a) In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body. (i) What is the probability that mathematics is being studied? (ii) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl? (iii) a boy? [8M]  
 b) Find the mean and variance of the Uniform distribution. [6M]

**Or**

- 6 a) Determine the discrete probability distribution, expectation, variance, S.D. of a discrete random variable X which denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once. [7M]  
 b) If z is normally distributed with mean 0 and variance 1, find [7M]  
 i)  $P(z \geq -1.64)$   
 ii)  $P(-1.96 \leq z \leq 1.96)$   
 iii)  $P(z \leq 1)$   
 iv)  $P(z \geq 1)$



## UNIT-IV

- 7 a) Explain t-distribution. [6M]
- b) In an air-pollution study performed at an experiment station, the following amount of suspended benzene soluble organic matter (in micrograms per cubic meter) was obtained for eight different samples of air: 2.2 , 1.8 , 3.1 , 2.0 , 2.4, 2.0 , 2.1, 1.2  
Assuming that the population sampled is normal, construct a 95% confidence interval for the corresponding true mean. [8M]

**Or**

- 8 a) A survey is proposed to be conducted to know the annual earnings of the old Statistics graduates of Delhi University. How large should the sample be taken in order to estimate the mean annual earnings within plus and minus Rs. 1,000 at 95% confidence level? The standard deviation of the annual earnings of the entire population is known to be Rs. 3,000. [7M]
- b) Assuming that the population standard deviation is 0.3, calculate the (i) 95% and (ii) 99% confidence intervals for the mean lead concentration in a river if the mean lead concentration recovered from a sample of lead measurements in 36 different locations is 2.6 gms/ml. [7M]

## UNIT-V

- 9 a) The length of life  $X$  of certain computers is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the null hypothesis that  $\mu = 800$  hours against the alternative that  $\mu = 800$  hours at 5% level of significance. [7M]
- b) Before an increase in excise duty on tea, 800 persons out of a sample of 1,000 persons were found to be tea drinkers. After an increase in duty, 800 people were tea drinkers in a sample of 1,200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty at 5% level? [7M]

**Or**

- 10 a) A random sample of 40 'geyers' produced by company A have a mean lifetime of 647 hours of continuous use with a S.D. of 27 hours, while a sample 40 produced by another company B have a mean lifetime of 638 hours with S.D. 31 hours. Does this substantiate the claim of company A that their 'geyers' are superior to those produced by company B at (i) 0.05 (ii) 0.01 L.O.S. [8M]
- b) An ambulance service company claims that on an average it takes 20 minutes between a call for an ambulance and the patient's arrival at the hospital. If in 6 calls the time taken (between a call and arrival at hospital) are 27, 18, 26, 15, 20, 32. Can the company's claim be accepted at 1% level ? [6M]

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UNIT-I

- 1 a) Show that for $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ the Cauchy-Riemann equations are satisfied at the origin but the derivative of $f(z)$ at origin does not exist. [10M]
- b) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circle $|z|=3$. [4M]

Or

- 2 a) Find the analytic function $f(z) = u + iv$ where $u = \sin x \cosh y$. [7M]
- b) Integrate $f(z) = x^2 + ixy$ from $A(1,1)$ to $B(2,8)$ along the curve $x=t, y=t^3$. [7M]

UNIT-II

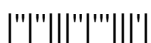
- 3 a) Find Taylor's expansion of $f(z) = \cos z$ about the point $z = \frac{\pi}{2}$. [6M]
- b) Apply the calculus of residues, evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$. [8M]
- Or**
- 4 a) Find the Laurent series of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$, for $|z| > 3$. [6M]
- b) Use residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + 1} dx$. [8M]

UNIT-III

- 5 a) A businessman goes to hotels X, Y, Z 20%, 50%, 30% of the time, respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbing. [7M]
 (i) Determine the probability that the businessman goes to hotel with faulty plumbing
 (ii) What is the probability that businessman's room having faulty plumbing is assigned to hotel Z?
- b) Find the mean and variance of the Normal distribution. [7M]

Or

- 6 a) Determine the discrete probability distribution, expectation, variance, S.D. of a discrete random variable X which denotes the maximum of the two numbers that appear when a pair of fair dice is thrown once. [7M]
- b) Out of 800 families with 5 children each, how many would you expect to have [7M]
 (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys. Assume equal probabilities for boys and girls.



UNIT-IV

- 7 a) Explain F-distribution. [7M]
- b) A cafe records that in $n = 81$ cases, the coffee beans for the coffee machine lasted an average of 225 cups with a standard deviation of 22 cups. Obtain a 90% confidence interval for μ , the population mean number of cups before the coffee machine needs to be refilled with beans. [7M]

Or

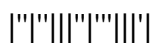
- 8 a) The mean muscular endurance score of a random sample of 60 subjects was found to be 145 with a s.d. of 40. Construct a 95% confidence interval for the true mean. Assume the sample size to be large enough for normal approximation. What size of sample is required to estimate the mean within 5 of the true mean with a 95% confidence? [7M]
- b) Explain Point estimation and Maximum error of estimate. [7M]

UNIT-V

- 9 a) A machine runs on an average of 125 hours/year. A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation 8.4 hours. Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance? [7M]
- b) A company has the head office at Calcutta and a branch at Bombay. The personnel director wanted to know if the workers at the two places' would like the introduction of a new plan of work and a survey was conducted for this purpose. Out of a sample of 500 workers at Calcutta, 62% favoured the new plan. At Bombay out of a sample of 400 workers, 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level ? [7M]

Or

- 10 a) To test the effects a new pesticide on rice production, a farm land was divided into 60 units of equal areas, all portions having identical qualities as to soil, exposure to sunlight etc. The new pesticide is applied to 30 units while old pesticide to the remaining 30. Is there reason to believe that the new pesticide is better than the old pesticide if the mean number of kgs of rice harvested / unit using new pesticide (N.P.) is 496.31 with s.d. of 17.18 kgs while for old pesticide (O.P.) is 485.41 kgs and 14.73 kgs. Test at a level of significance (i) $\alpha = 0.05$ (ii) 0.01. [8M]
- b) In a random sample of 10 bolts produced by a machine the mean length of bolt is 0.53 mm and standard deviation 0.03 mm. Can we claim from this that the machine is in proper working order if in the past it produced bolts of length 0.50 mm? Use (i) 0.05 (ii) 0.01 L.O.S. [6M]



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UNIT-I

- 1 a) Show that $f(z) = \begin{cases} \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic at $z = 0$, although the [10M]

Cauchy-Riemann equations are satisfied at the origin

- b) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^z}{z-2i} dz$ where C is the circle [4M]
 $|z-2i| = 4$.

Or

- 2 a) Find the analytic function $f(z) = u + iv$ where $u = e^x \cos y$. [7M]
 b) Evaluate $\int_0^{1+i} (x^2 + iy) dz$ along the paths $y = x$ and $y = x^2$. [7M]

UNIT-II

- 3 a) Find Taylor's expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about the point $z = i$. [6M]
 b) Use residue theorem to evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos \theta}$. [8M]

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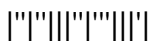
- 4 a) Find the Laurent series of $f(z) = \frac{1}{z^2 - 4z + 3}$, for $1 < |z| < 3$. [6M]
 b) Apply the calculus of residues, to evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$. [8M]

UNIT-III

- 5 a) Companies B_1, B_2, B_3 produce 30%, 45% and 25% of the cars respectively. It is known that 2%, 3% and 2% of the cars produced from B_1, B_2 and B_3 are defective. [7M]
 (i) What is the probability that a car purchased is defective?
 (ii) If a car purchased is found to be defective what is the probability that this car is produced by company B_3 ?
 b) Find the mean and variance of the Poisson distribution. [7M]

Or

- 6 a) Determine the discrete probability distribution, expectation, variance, S.D. of a discrete random variable X which denotes the sum of the two numbers that appear when a pair of fair dice is thrown once. [7M]
 b) Determine the probability of getting 9 exactly twice in 3 throws with a pair of fair dice. [7M]



UNIT-IV

- 7 a) Explain χ^2 -distribution. [7M]
b) The dean of a college wants to use the mean of a random sample to estimate the average amount of time students take to get from one class to the next, and she wants to be able to assert with 99% confidence that the error is at most 0.25 minute. If it can be presumed from experience that $\sigma = 1.40$ minutes, how large a sample will she have to take? [7M]

Or

- 8 a) Define Point and Interval estimations. [7M]
b) If on the average, the test strips painted across heavily travelled roads in 15 different locations, disappeared after they had been crossed by 146692 cars with s.d. 14380 cars, calculate 99% confidence intervals for the true average number of cars it takes to wear off the paint, assuming normal population. [7M]

UNIT-V

- 9 a) A manufacturer of tyres guarantees that the average lifetime of its tyres is more than 28000 miles. If 40 tyres of this company tested, yields a mean lifetime of 27463 miles with S.D. of 1348 miles, can the guarantee be accepted at 0.01 L.O.S.? [7M]
b) Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal, are same against that they are not, at 5% level. [7M]

Or

- 10 a) In a survey of buying habits, 400 women shoppers are chosen at random in super market 'A' located in a certain section of the city. Their average weekly food expenditure is Rs. 250 with a standard deviation of Rs. 40. For 400 women shoppers chosen at random in super market 'B' in another section of the city, the average weekly food expenditure is Rs. 220 with a standard deviation of Rs. 55. Test at 1% level of significance whether the average weekly food expenditure of the two populations of shoppers are equal. [7M]
b) An auditor claims that he takes on an average 10.5 days to file income tax returns (I.T. returns). Can this claim be accepted if a random sample shows that he took 13, 19, 15, 10, 12, 11, 14, 18 days to file I.T. returns? Use (i) 0.01 (ii) 0.05 L.O.S. [7M]

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UNIT-I

- 1 a) Show that $f(z) = \sqrt{|xy|}$ is not analytic at $z = 0$, although the Cauchy-Riemann equations are satisfied at the origin. [8M]
- b) Integrate $f(z) = x^2 + ixy$ from $A(1, 1)$ to $B(2, 4)$ along the curve $x = t, y = t^2$. [6M]

Or

- 2 a) Are the following function harmonic? If your answer is yes, find a corresponding analytic function $f(z) = u(x, y) + i v(x, y)$. Where $u(x, y) = e^{-x} \sin 2y$. [7M]
- b) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ where C is the circle $|z| = 4$. [7M]

UNIT-II

- 3 a) Find Taylor's expansion of $f(z) = \cos z$ about the point $z = \frac{\pi}{4}$. [6M]
- b) Use residue theorem to evaluate $\int_0^{2\pi} \frac{d\theta}{7 + 6 \cos \theta}$. [8M]

Or

- 4 a) Find the Laurent's series of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ about $z = 0$ in the region $2 < |z| < 3$. [6M]
- b) Use residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 9)}$. [8M]

UNIT-III

- 5 a) Box I contains 1 white, 2 red, 3 green balls, Box II contains 2 white, 3 red, 1 green balls, Box III contains 3 white, 1 red, 2 green balls. Two balls are drawn from a box chosen at random. These are found to be one white and one red. Determine the probability that the balls so drawn came from box II. [7M]
- b) Find the mean and variance of the Binomial distribution. [7M]

Or

- 6 a) Suppose a continuous R.V. x has the probability density [7M]
- $$f(x) = \begin{cases} k(1-x^2), & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$
- (i) Find k (ii) Find $P(0.1 < x < 0.2)$ (iii) $P(x > 0.5)$.
- b) The average number of phone calls/minute coming into a switch board between 2 and 4 PM is 2.5. Determine the probability that during one particular minute there will be (i) 0 (ii) 1 (iii) 2 (iv) 3. [7M]

UNIT-IV

- 7 a) Define Population and Sample with example. [7M]
b) A random sample of size $n = 100$ is taken from a population with $\sigma = 5.1$. Given that the sample mean is $\bar{x} = 21.6$, construct a 95% confidence interval for the population mean μ . [7M]

Or

- 8 a) Construct a 99% confidence interval for the true mean weight loss if 16 persons on diet control after one month had a mean weight loss of 3.42 kgs with s.d. of 0.68 kgs. [7M]
b) For an F -distribution find [7M]
i). $F_{0.05}$ with $v_1 = 7$ and $v_2 = 15$
ii). $F_{0.01}$ with $v_1 = 24$ and $v_2 = 19$
iii). $F_{0.95}$ with $v_1 = 19$ and $v_2 = 24$
iv). $F_{0.99}$ with $v_1 = 28$ and $v_2 = 12$.

UNIT-V

- 9 a) In a random sample of 100 tube lights produced by company A, the mean lifetime of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of (i) 0.05 (ii) 0.01? [8M]
b) In a sample of 1,000 people in Maharashtra, 540 are rice eaters and the rest are wheal eaters. Can we assume that both rice and wheat are equally popular in this State at 1% level of significance? [6M]

Or

- 10 a) Mean lifetime of computers manufactured by a company is 1120 hours with standard deviation of 125 hours. Test the hypothesis that mean lifetime of computers has not changed if a sample of 8 computers has a mean lifetime of 1070 hours .Use (i) 0.05 (ii) 0.01 L.O.S. [6M]
b) A study of TV viewers was conducted to find the opinion about the mega serial 'Ramayana'. If 56% of a sample of 300 viewers from south and 48% of 200 viewers from north preferred the serial, test the claim at 0.05 L.O.S. that (i) there is a difference of opinion between south and north (ii) 'Ramayana' is preferred in the south. [8M]

